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HIERARCHICAL MODULAR STRUCTURES AND THEIR GEOMETRICAL CONFIGURATIONS FOR FUTURE LARGE SPACE SYSTEMS

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SUMMARY

Presently it takes too much time and cost to construct large space systems such as the International Space Station. Sometimes we must modify or alter the mission and configuration of such system as required to comply not only with technical but also social or political affairs. Therefore basic concepts for future large space systems need to be able to adjust to such various changes. Also the life cycles of such space structural systems need to take account of such change. In this paper, hierarchical modular structure systems suitable for such future structures are proposed. These systems include structural shapes with properties of fractals. The proposed structural systems are composed of identically shaped modules which are hierarchically assembled using systematic rules. They can systematically form structures of various shapes and sizes. The geometrical aspects of modular space structures and two-dimensional and three-dimensional examples of hierarchical modular structural systems based on symmetry group are described.

Keywords: *Large Space Structures, Hierarchical Systems, Modular Structures.*

1. INTRODUCTION

Generally it takes too much time and cost to construct large and complex space systems seen in the case of the International Space Station. During construction of such structures, some changes of initial designs or schemes are inevitable. Reasons for the changes are not only technical but also social or political affairs. Therefore, basic design principles for future large space structures need to be able to adjust to such changes.

Adaptation to environmental changes means temporal transition of structural configurations from one phase to another. Environmental changes involve boundary conditions, applied forces, sizes, and shapes of structures. Temporal transition is a basic aspect of a life cycle concept. A life cycle of an artificial structure consists of all phases and temporal transitions from "to be planned" to "to be reused". Research on life cycle engineering of industrial goods and buildings on ground has developed remarkably. Recently research on distribution services and production systems has

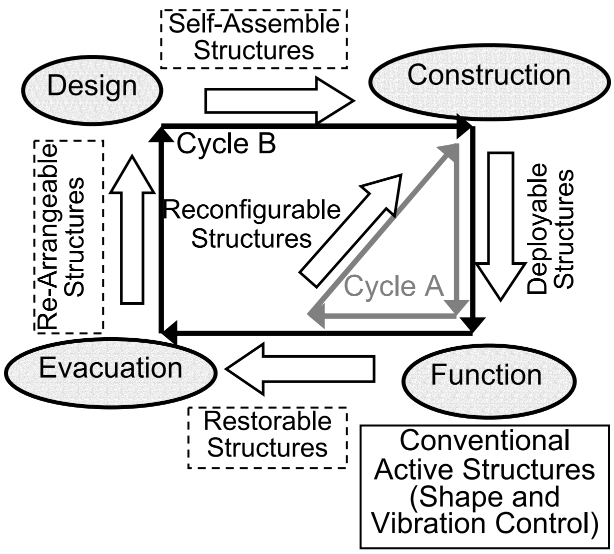


Figure 1. *A Life Cycle for Future Space Structure Systems*

also been carried out [1]. But in this research, human activities instead of structural functions change the phases of their life cycles, because it is easy to directly access and support them. On the other hand, space structures are not easily accessible. Therefore, space structures must have functions which can change the phases of their cycles. Figure 1 shows a life cycle model of future space systems with such functions. There have been few studies concerning life cycle approach to space structures [2].

In this paper, we propose hierarchical modular structures for future space systems, which include the structures with fractal properties. The proposed structures consist of a number of modules with identical shapes which are hierarchically assembled. They can form various sizes and shapes with the same-shaped modules assembled by systematical rules. We also introduce assembly rules based on symmetric group theory. Such rules easily generate hierarchically symmetric structures. Two-dimensional and three-dimensional examples are demonstrated in this paper.

2. CONCEPT AND BACKGROUND OF HIERARCHICAL MODULAR STRUCTURES

Adaptive structures can adjust to environmental changes which may include changes to shapes, sizes, boundary conditions and applied forces. The adjustability to environmental changes is one of typical features of things in nature. One of the reasons that structural systems of things in nature are able to efficiently adapt to environmental changes could be their hierarchical systems, since hierarchical systems are able to create sufficient variety using limited resources and assembly rules. Typical examples of hierarchical structures in nature are structures with fractal properties [3]. Some mechanical properties of representative truss structures with fractal properties are described in [4]. The results indicate some advantages for large or complex structures from the viewpoint of adjustability to environmental changes.

Figure 2 shows a concept of hierarchical modular structures proposed in this paper. The concept of hierarchical modular structures is a combination of hierarchical structures in nature and current modular structures. They consist of a number of identically shaped modules, which are hierarchically assembled. Systematical assembly rules are expressed in the following equations. The assembly rules give hierarchical modular structures useful geometrical properties: a) geometrical symmetry, b) extendibility of shapes and sizes, and c) regular openings.

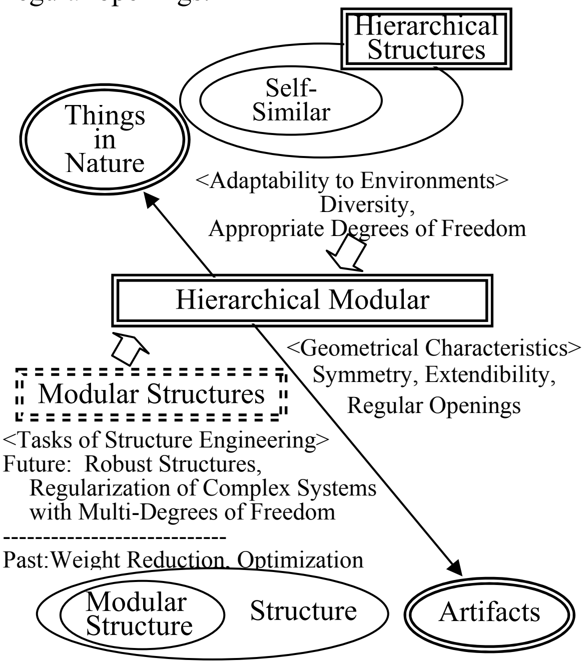


Figure 2. *Concept of Hierarchical Modular Structures*

$$\begin{aligned}
\boldsymbol{G}^1 &= \boldsymbol{A}^1 (\boldsymbol{M}, \boldsymbol{M}, \ldots, \boldsymbol{M}),\\
\boldsymbol{G}^2 &= \boldsymbol{A}^2 (\boldsymbol{G}^1, \boldsymbol{G}^1, \ldots, \boldsymbol{G}^1)\\
&\vdots\\
\boldsymbol{G}^k &= \boldsymbol{A}^k (\boldsymbol{G}^{k-1}, \boldsymbol{G}^{k-1}, \ldots, \boldsymbol{G}^{k-1})\\
&= \boldsymbol{A}^k (\boldsymbol{A}^{k-1}(\boldsymbol{G}^{k-2}, \ldots, \boldsymbol{G}^{k-2}), \ldots, \boldsymbol{A}^{k-1}(\boldsymbol{G}^{k-2}, \ldots, \boldsymbol{G}^{k-2}))\\
&= \ldots
\end{aligned}$$

Equation 1

where $\boldsymbol{M}$ is an initial member, $\boldsymbol{A}^k$ is kth assembly rule to generate a next-generation structure and $\boldsymbol{G}^k$ is a kth-generation structure. This expression shows that one $\boldsymbol{G}^k$ is composed of some previous generations $\boldsymbol{G}^{k-1}$ s. In particular, a uniform $\boldsymbol{A}^k$ generate the structures with fractal properties.

The hierarchical modular structures have the same advantages as the current modular structures like the Japanese engineering test satellite ETS-VIII [5]. For example, it is easy to produce or test each one module of the structure, to transport the modules in the limited space like a rocket cargo, and to analyze mechanical behavior using some sub-structuring methods. However, the current modular structures have some disadvantages: duplicated members increase their weight, and their systems are too complicated due to their multiple degrees of freedom. The geometrical properties of hierarchical modular structures make it possible to eliminate these disadvantages. In the following some geometrical configurations of hierarchical modular structures are shown and discussed.

3. TWO-DIMENTIONAL GEOMETRICAL CONFIGURATIONS OF HIERARCHICAL MODULAR STRUCTURES

Hierarchical modular structures consist of a number of systematically assembled of identically shaped modules. Let us specify 2-dimensional assembly rules. Let assembly rules $\boldsymbol{A}^k$ in equation (1) be replaced by closed-loop configurations derived from rotation mappings. Closed-loop configurations have some advantages over serial or branching configurations from the viewpoint of structural engineering, as they can easily form structures with geometrical symmetry and higher stiffness. They can also reduce influence from local damage or failure of their members. The nature of the configurations is best illustrated by means of the following examples.

One basic module of regular a n-gon shape is generated by rotation mapping through an angle $2\pi/n$ about a fixed point on the symmetry axis of the one dimensional member (Fig.3 (a) ; $n = 3$). A second generation is generated by rotation mapping through an angle $2p/m$ about a fixed point on the symmetry axis of the first-generation (Fig.3 (b); $m = 3$). Considering rotation mappings and axes, we define mathematical expressions of a first-generation and second generation as n_0^1 and $m_{q^2}^2(n_0^1)$, respectively. q indicates the symmetry axis (arrows in Fig.3). A third generation is generated by rotation mapping through an angle $2p/1$ about a fixed point on the symmetry axis of the second generation (Fig.3 (c); $1 = 3$). We express this third-generation as $1_{q^3}^3\left(m_{q^2}^2\left(n_0^l\right)\right)$. In the same way, we can generate and express following generations. In particular, $3_p^3\left(3_p^2\left(3_0^1\right)\right)$shown in Fig.3(c) is known as the third-generation Sierpinski-gasket, which is one of the famous shapes with fractal properties. This indicates that our hierarchical modular structures include other structures with fractal properties.

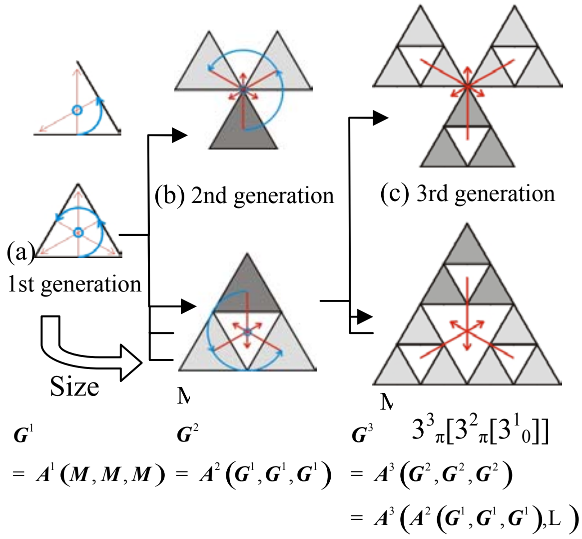


Figure 3. *Examples of Composition of Hierarchical Modular Structures*

Figures 4 and 5 demonstrate various 4th-generation models, which consist of 54 hexagonal modules. These plane examples demonstrate that the proposed configurations provide the three geometrical properties mentioned before (geometrical symmetry, extendibility of shapes and sizes and regular openings). Moreover this figure indicates that our method can be applied starting from a finite number of modules or from an overall structural shape. This property is helpful when designing artificial structures. In contrast, the conventional expressions like the Schlafli symbol represent only local relationship between polygon edges and faces.

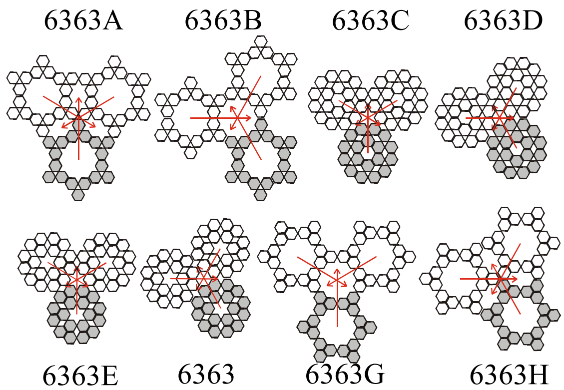


Figure 4. *Examples of Hierarchical Modular Structures Composed of Hexagonal Modules (Type 6363)*

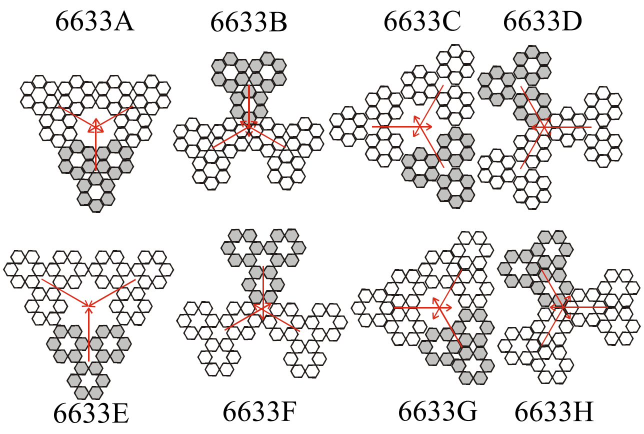


Figure 5. *Examples of Hierarchical Modular Structures Composed of Hexagonal Modules (Type 6633)*

Figure 6 (a) shows first mode frequencies of frame models (4th generations from Fig.4). In this case, connective members between modules are considered. Total number of the connective members (Fig.6 (b)) and total weight (Fig. 6 (c)) are also shown. The figure indicates that the proposed configurations, because of their variety of shapes, can describe various hierarchical modular structures with various mechanical properties.

4. GEOMETRICAL CONFIGURATIONS OF THREE-DIMENTIONAL HIERARCHICAL MODULAR STRUCTURES

The generalization of our configuration of hierarchical modular structures into three dimensions is investigated in this section. Rotation mapping must be extended to the finite rotation group in Euclidean 3-dimensional space [6]. Substituting assembly rules based on three-dimensional group into the assembly rules A^k in equation 1 gives us 3-dimensional hierarchical modular structures. Finite rotation group involves the three subgroups: the cyclic group, the dihedral subgroup, and the polyhedron group. Let us specify the assembly rules in 3D space. The nature of our method is best illustrated by means of examples based on each subgroup. Here we treat a regular polyhedron as a basic module of first generation.

Figure 7 shows second and third generations with tetrahedral modules based on the cyclic group. This pattern is the so-called Waxman type of octahedral plane truss structures. In this case, using projection a spatial configuration is simplified to a plane configuration. Figure 8(a) shows three projections of a basic tetrahedron with its symmetrical plane. These three projections are derived from three directions in Fig.8(b), respectively. Replacing regular polygon by these projections and replacing symmetrical axis by the projection of symmetrical plane, plane configurations mentioned in the previous section can be applied to spatial configurations. This configuration can be applied to any other polyhedron modules.

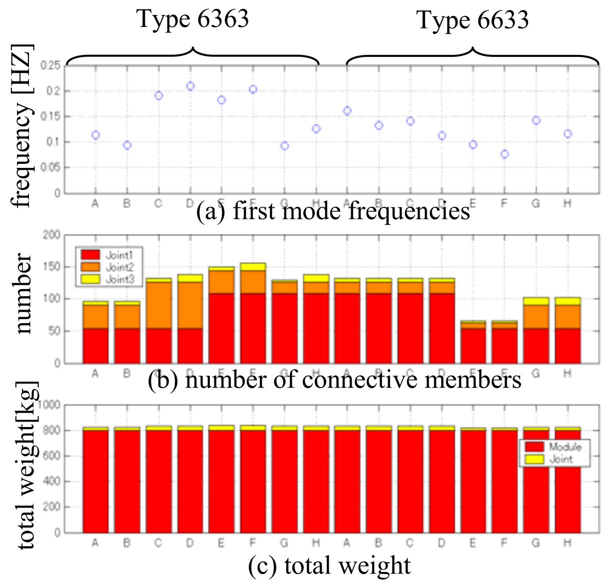


Figure 6. *First Mode Frequencies of Frame Structures of Type 6363 and Type 6633*

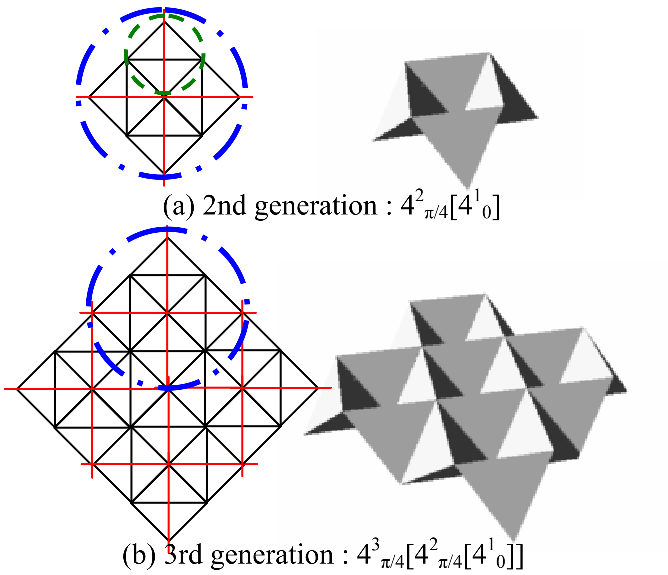
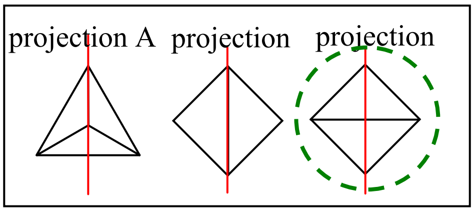
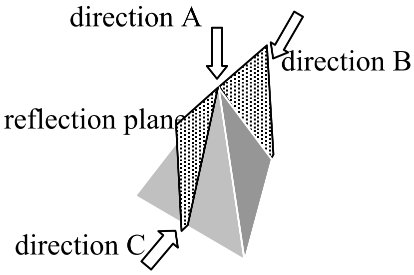


Figure 7. *2nd and 3rd examples based on the cyclic groups*



(a) projections of a tetrahedron



(b) directions of

Figure 8. *Projection for 3-dimensional hierarchical modular structures based on the cyclic groups*

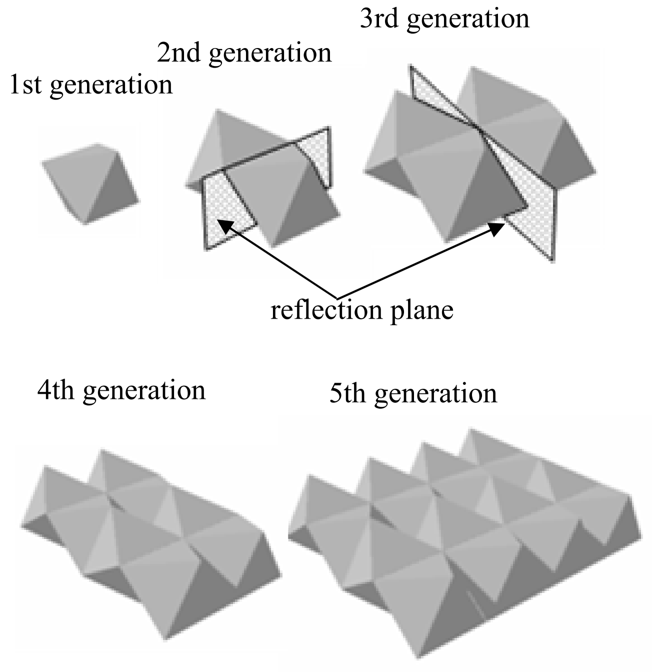
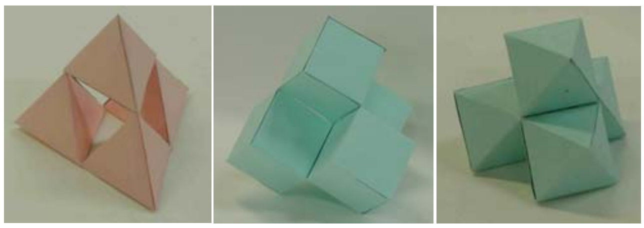


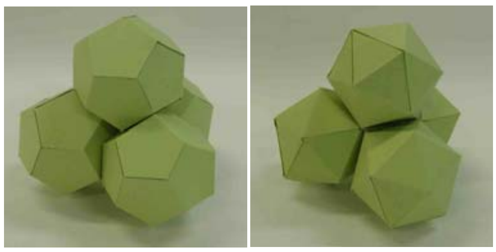
Figure 9. *3-dimensional hierarchical modular structures based on the dihedral groups*

Figure 9 shows a configuration with octahedral modules based on the dihedral group. Starting from a first generation, iterated reflections can provide the following generations. This pattern also leads to another type of octahedral plane truss structures.

In the polyhedron group symmetry of the vertexes is found in the regular polyhedron. An expression of a first generation shaped tetrahedron is 4_t^1, which means that 4 vertexes form the tetrahedron symmetry. By the same rule, expressions of cube, octahedron, dodecahedron, and icosahedron are 8_c^1, 6_o^1, 20_d^1, and 12_i^1, respectively. Next, for example, $4_t^2(4_t^1)$ indicates a second generation which consists of 4 tetrahedrons placed on each vertex of a larger tetrahedron. Figure 10 illustrates second generations produced by mapping $4_t^2[\ ]$ based on the tetrahedral symmetry group. Repeating this operation produces the next generation. Two examples of third generations produced by mapping $4_t^3[\ ]$are shown in Fig.11. These configurations can be applied to the other regular polyhedral symmetry groups. Table 1 show second generations derived from various mappings and polyhedral first generations. In particular, second generations marked # in Table 1 are the three dimensional hierarchical structures with fractal properties.

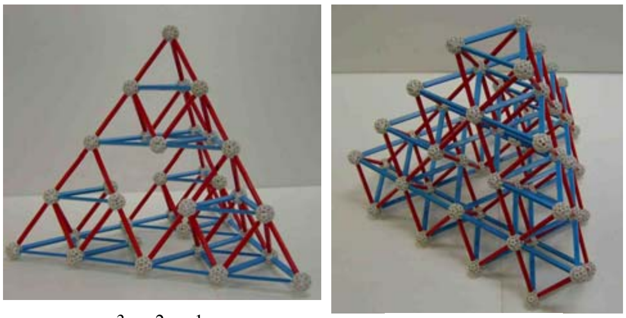


$4^2{}_t[4^1{}_t]$ $4^2{}_t[8^1{}_c]$ $4^2{}_t[6^1{}_o]$



$4^2{}_t[20^1{}_d]$ $4^2{}_t[12^1{}_i]$

Figure 10. *Second generation models of 3-dimensional structure generated by mapping* $:4^2{}_t[]$

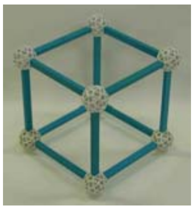


(a) $4^3_t[4^2_t[4^1_t]]$ (b) $4^3_t[4^2_t[6^1_o]]$

Figure 11. *3rd-generation models of 3-dimensional structures*

Table.1 *Second Generation of Three Dimensional Hierarchical Modular Structures based on regular polyhedron group*

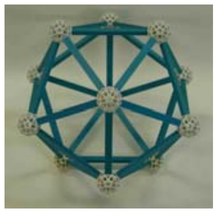
tetrahedron



cube

octahedron

dodecahedron



icosahedron

1st generation / mapping for 2nd generation	4^1_t: tetrahedron	8^1_c: cube	6^1_o: octahedron	20^1_d: dodecahedron	12^1_i: icosahedron
4^2_t[ ]: tetrahedron	$4^2_t[4^1_t]$ *#	$4^2_t[8^1_c]$*	$4^2_t[6^1_o]$*	$4^2_t[20^1_d]$ *	$4^2_t[12^1_i]$ *
8^2_c[ ]: cube	$8^2_c[4^1_t]$	$8^2_c[8^1_c]$#	$8^2_c[6^1_o]$	$8^2_c[20^1_d]$	$8^2_c[12^1_i]$
6^2_o[ ]: octahedron	$6^2_o[4^1_t]$	$6^2_o[8^1_c]$	$6^2_o[6^1_o]$#	$6^2_o[20^1_d]$	$6^2_o[12^1_i]$
20^2_d[ ]: dodecahedron	$20^2_d[4^1_t]$	$20^2_d[8^1_c]$	$20^2_d[6^1_o]$	$20^2_d[20^1_d]$#	$20^2_d[12^1_i]$
12^2_i[ ]: icosahedron	$12^2_i[4^1_t]$	$12^2_i[8^1_c]$	$12^2_i[6^1_o]$	$12^2_i[20^1_d]$	$12^2_i[12^1_i]$ #

** : Mapping for 2nd generaion is based on a tetrahedoran(see Fig. 10)*
#: Relation between mapping and mapped polyhedron is self-similar

The model expressed as $4^3_t\left(4^2_t\left(4^1_t\right)\right)$ (Fig.10(a)) is equal to a third generation of spatial Sierpinski-gasket. We can generate a k-th generation expressed as $1^k_{polygon(k)}(m^{k-1}_{polygon(k-1)}(\ldots(n^1_{polygon(1)})))$, which provides a k-dimensional table of k-th generations. These spatial examples also prove that the proposed configurations have the important geometrical properties mentioned before: geometrical symmetry, extendibility of shapes and sizes, and regular openings. In the three dimensional case, our configurations can start from a finite number of modules or from an overall structural shape, since the shape expressions can explicitly indicate the number and shape of each generation. This property is useful when designing artificial structures. In contrast, the conventional expressions like Shlafli symbols and the Miller indices in crystallographic study [7] represent only a local relationship between polyhedral edges and faces.

5. CONCLUSION

This paper shows a concept and geometrical configurations of hierarchical modular structures, which consist of systematically assembled identically shaped modules. The concept of hierarchical modular structures is inspired by hierarchical structures in nature which have the capability to adjust to environmental changes. The adaptation to environmental changes will be required for future large space structures from the viewpoint of their autonomous life cycles.

Also shown were geometrical configurations of 2-dimensional and 3-dimensional hierarchical modular structures. Geometrical configurations of 2-dimensional hierarchical modular structures are hierarchical closed-loops of polygonal modules. The assembly rule is based on a rotation mapping in plane. The structures generated from these configurations have important and useful geometrical properties: geometrical symmetry, extendibility of sizes and shapes, and regular openings. These properties can eliminate some drawbacks of the current modular structures and provide some advantages for future large space structures. The proposed configurations include shapes which have properties of fractals.

Geometrical configurations of 3-dimensional hierarchical modular structures are hierarchically assembled regular polyhedral modules. The assembly rule is based on the finite rotation group in 3-dimensional Euclidean space, which is equivalent to rotation mapping in 2-dimentional plane. The finite rotation group includes the cyclic subgroup, the dihedral subgroup, and the polyhedron subgroup. Some examples generated by mapping based on each subgroup are shown. They prove that these 3-dimensional configurations also have the same geometrical properties as those of the 2-dimensional case. The proposed configurations again also include shapes which have properties of fractals.

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