Control and Mechanical Characteristics of Hierarchical Modular Structures

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Future space structure systems need adaptability to various changes from the viewpoint of total cost for their missions. In order to introduce such adaptability into artificial structure systems, hierarchical modular structures are proposed and studied. They consist of a number of basic modules, which are assembled with hierarchical assembly rules. And each module is supposed to have autonomous functions based on localized simple control laws. Modular structures with such active functions are regarded as autonomous decentralized systems. To keep independency of each module and to regulate interactions between modules, active connective members are introduced to modular structures. We especially focus on a role of the active connective members in this paper. Examples of vibration suppression of some one-dimensional and two-dimensional models of hierarchical modular structures are investigated. Various control strategies are evaluated from the viewpoint of control energy. And it is shown that introducing connective members and hierarchical configuration are able to systematically provide various control strategies. Lastly we discuss static mechanical characteristics of two-dimensional models of hierarchical modular structure from the viewpoint of active control strategy.

Nomenclature

\[ k_{i}^{m} = \text{stiffness sub-matrix of an } i\text{-th module} \]
\[ k_{i,j}^{e} = \text{stiffness sub-matrix of a connective between } i\text{-th and } j\text{-th modules} \]
\[ x_{i}^{m} = \text{state variable of an } i\text{-th module} \]
\[ x_{i,j}^{e} = \text{state variable of a connective between } i\text{-th and } j\text{-th modules} \]
\[ f_{i}^{m} = \text{forces on an } i\text{-th module} \]
\[ f_{i}^{m\text{--external}} = \text{external forces on an } i\text{-th module} \]
\[ f_{i}^{m\text{--control}} = \text{control forces on an } i\text{-th module} \]
\[ f_{i,j}^{e} = \text{forces on a connective between } i\text{-th and } j\text{-th modules} \]
\[ f_{i}^{e\text{--external}} = \text{external forces on a connective between } i\text{-th and } j\text{-th modules} \]
\[ f_{i}^{e\text{--control}} = \text{control forces on a connective between } i\text{-th and } j\text{-th modules} \]
\[ \alpha_{i}^{m}, \beta_{i}^{m} = \text{design variable for direct displacement and velocity feedback (DDVF) control on an } i\text{-th module} \]
\[ \alpha_{i,j}^{e}, \beta_{i,j}^{e} = \text{design variable for DDVF control on a connective between } i\text{-th and } j\text{-th modules} \]
\[ \gamma_{i,j} = \text{design variable for separation control on a connective between } i\text{-th and } j\text{-th modules} \]
\[ \sigma_{i,j}^{e} = \text{internal forces on a connective between } i\text{-th and } j\text{-th modules} \]

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I. Introduction

missions for space developments have aimed at various and precise tasks recently. Space structures for such missions have become so large and complicated that their constructions take too much time and cost as seen in the case of the International Space Station. During construction of such structures, some changes of initial designs or schemes are unpreventable. They come both from engineering developments due to new technological or environmental aspects, and from social transitions due to new political or economical aspects. Future space structure systems are desired to have adaptability to these various changes.

From the viewpoint of adaptability, various researches on adaptive structures have been made from the 1980’s\(^1\). A considerable number of studies have been conducted on optimum and robust control theories\(^2\). Concerns with adaptive structures used piezoelectric material s and shape memory alloys have been widely studied\(^3\). These researches usually aim to find special solutions adapted to special environment. However, mostly the solutions could not adapt to other environments. Then to adapt more wide environments and their changes, we proposed a concept of hierarchical modular structure systems\(^4\). Proposed structures are composed of a number of basic modules with autonomous active functions, and they are constructed by systematic assembly rules similar to those of fractal formations. They show preferable characteristics of symmetry, extendibility, and regular openings.

In this paper we provide control characteristics of hierarchical modular structures. Also we discuss their mechanical characteristics including maximum values and distributions of internal forces from the viewpoint of active structures.

II. Hierarchical Modular Structures and Autonomous Decentralized Systems

A. Mathematical Expression and Examples of Hierarchical Modular Structures

Hierarchical modular structures can be mathematically expressed by following simple equations.

\[
G^1 = A^1(M, M, \cdots, M), \\
G^2 = A^2(G^1, G^1, \cdots, G^1) \\
\vdots \\
G^k = A^k(G^{k-1}, G^{k-1}, \cdots, G^{k-1}) \\
= A^k\left(A^{k-1}(G^{k-2}, \cdots, G^{k-2}), \cdots, A^{k-1}(G^{k-2}, \cdots, G^{k-2})\right) \\
\vdots
\]

where \(M\) is an initial member, \(A^k\) is \(k\)th assembly rule to generate a next-generation structure, and \(G^k\) is a \(k\)th-generation structure. \(G^1\) is defined as a basic module in this paper. This expression shows that one \(G^k\) is composed of some previous generations \(G^{k-1}\)’s. It should be noted that this algorithm can directly apply to analysis and control of structures. In the case of self-similar structures, all assembly rules are the same.

Figure 1 shows some planer examples in the case that \(M\) is an axial member and a basic module is hexagonal in

![Figure 1. Planer examples of 4th generation hierarchical modular structures based on hexagons.](image)
planer shape. To keep geometrical symmetry, we introduce assembly rules \( A \) based on rotational mappings. The notation of “Type 6363” or “Type 6633” comes from how to assemble modules. These sixteen patterns have the same fifty-four basic modules.

**B. Elements and Interactions between Elements of Autonomous Decentralized Systems**

Autonomous decentralized systems are composed of number of elements with autonomous active functions. When the active functions are simple and based on localized information, such decentralized systems are more suitable for large or complicated systems than centralized systems. Because it takes too much cost for centralized systems to identify mathematical models and solve inverse kinematics of large or complicated systems. Furthermore, according to a concept of centralized systems we must accumulate errors and time-lags. Concept of autonomous decentralized systems is one solution for these problems.

Generally total performance of autonomous decentralized systems depends on both autonomous ability of each element and interactions between such autonomous elements. Current researches on autonomous decentralized systems mainly focus on software systems including power systems, communication systems. Elements of such software systems are supposed to be easily connected or disconnected each other. Then we can keep independency of each element and easily regulate interactions between elements. On the contrary, elements of structure systems based on hardware systems cannot be connected or disconnected each other so easily. When we introduce autonomous decentralized systems into structural engineering, we must discuss autonomous ability of each element and interactions between elements from the viewpoint of structural engineering.

We classify systems with autonomous elements and their physical connections between elements (Fig.2). In structural engineering, autonomous ability of each element means local control ability of each element based on local information. And interactions among elements mean transmissions of mechanical information including displacements, internal forces besides communication. If there is no connection between elements (Fig.2 (a)), they do not form a structure system. If all degrees of freedom between elements are completely fixed (Fig.2 (b)), independency of each element must be lost and we do not explicitly treat interactions between elements. If system has some mechanisms (Fig.2(b) and (c)), such structure systems are statically unstable. Only when connective members are introduced (Fig.2 (d) and (e)), each module keeps some independency and we can explicitly treat interactions between elements. Especially active connective (Fig. 2 (e)) is able to regulate interactions between active elements. Then we focus on the systems introduced active connectives in this paper.

![Figure 2. Physical Connections between Autonomous Elements](image)

C. Introducing Connective Members into Hierarchical Modular Structures

1. **Active Connective Members Explicitly Transmit and Regulate Mechanical Interactions between Modules.**

When a module of hierarchical modular structures has autonomous functions, hierarchical modular structures are regarded as one of autonomous decentralized systems. However, in order to treat mechanical interactions between
active modules, we need to introduce connective members between modules mentioned above. Mechanical interactions are transmissions of mechanical information including stress and displacements.

Using mathematical expression of equation of motion, we show this role of connective members. Eq. (2) is equation of motion concerning modular structures shown in Fig. 2 (e). In this equation, sub-matrix, state variable, and control forces concerning connectives are explicitly expressed. It means that we can easily regulate mechanical interactions between active modules.

\[
M_x + K_x = f_x
\]

Where,

\[
K_x = \begin{cases} 
  k^m_1 & 1 \\
  k^m_i & 1,2,3,\ldots, n \\
  k^m_{n-1} & 1,2,3,\ldots, n \\
  0 & \text{otherwise} 
\end{cases}
\]

\[
x = [x^m_1, x^m_2, x^m_3, \ldots, x^m_{n-1}, x^m_n]^T
\]

\[
f = [f^m_1, f^m_2, f^m_3, \ldots, f^m_{n-1}, f^m_n]
\]

Using mathematical expression of equation of motion, we show this role of connective members. Eq. (2) is equation of motion concerning modular structures shown in Fig. 2 (e). In this equation, sub-matrix, state variable, and control forces concerning connectives are explicitly expressed. It means that we can easily regulate mechanical interactions between active modules.

Figure 3. Arguments and functions of control forces and connection topology 1.
(current approach of autonomous decentralized systems)
2. Modular Structures with Connective Members are Robust over Changes of Connection Topology.

Introducing connective members, we can unify expressions of control forces despite of connective topology. An element of usual autonomous decentralized systems, including cellular automata, decides their control forces for next situation due to information from adjacent elements (Fig. 3). In connection topology A, number of arguments in interior region (a) is different from that in boundary region (b). In connection topology B, number of arguments in interior region (a) is different from that in another interior region (a’). It results that we must change functions for control forces as to changes of connection topology or position of module, according to usual concept of autonomous decentralized systems.

According to proposed concept with connective members, number of arguments for control forces is independent from connection topology or position of module (Fig 4). Number of connective members varies as to changes of connection topology or position of module, instead of number of arguments. It results that we do not change mathematical function for control forces according to this concept. This is effective for systems with changes of shapes or sizes. That is, modular structures with connective members are robust over changes of connection topology. We also unify both hardware and software of controllers, because their design is based on mathematical expression of control forces.
III. Control Strategies of One-Dimensional Hierarchical Modular Structures

A. Spring Mass Models and their Hierarchical Configurations

We present two models of one-dimensional hierarchical modular structures to evaluate a role of connective members; spring mass model and model with rigid body modules. Figure 5 shows a spring mass model analyzed here. A module is assumed to be composed of two masses and a spring with a damper. This spring represents elastic property of a module structure. A connective is assumed to be composed of one mass and two springs with damper. These springs represent both elastic property of connective structure and connection property between module and connectives, including friction, gap.

Mass of a module is 1.0, and stiffness of spring of a module is also 1.0. Using parameter ratio \(lc\), mass of connective is \(lc \times 2.0\), and stiffness of spring of a connective is \(2.0/lc\). In this paper, \(lc\) is assumed to be 0.001, 0.01, and 0.1. Number of modules is 15. Structural damping is assumed and boundary condition is fix-free.

Figure 6 shows tow patterns of hierarchical configurations of spring mass models (number of module = 15). Second and third generation of configuration A are composed of five first generations (basic modules) and three second generations, respectively. Second and third generation of configuration B are composed of three first generations and five second generations, respectively. We can directly apply such hierarchical configurations to systematically construct various control strategies as follows.

![Figure 5. Spring mass model of modular structures.](image)

![Figure 6. Examples of hierarchical configurations of the spring-mass model.](image)
B. Systematical Construction of Various Control Strategy and Numerical Simulation Results

Control laws must be simple and based on local information. In this paper, three following control laws are used. Control law on modules is local LQR control, which can be expressed by eq.(3). Its design parameters are calculated from Riccati equation of small size system such as system with only two modules.

\[ f_i^{\text{m-control}} = \alpha_i^m x_i^m + \beta_i^m x_i^m \]  

(3)

Two control laws on connective is presented. One is LQR control, which can be expressed by eq.(4), and one is so-called separation control, which can be expressed by eq.(5). Design parameters of the former law are calculated from Riccati equation of small size system mentioned in case of control law on module. Design parameters of the latter law are decided optionally. Separation control is aiming to prevent interactions between modules.

\[ f^c_{j_1,j_2} = \alpha^c_{j_1,j_2} x_{j_1,j_2}^c + \beta^c_{j_1,j_2} x_{j_1,j_2}^c \]  

(4)

\[ f^c_{j_2} = -\gamma^c_{j_1,j_2} \sigma^c_{j_1,j_2} \]  

(5)

Table 1 shows some control strategies combined of these simple control laws for spring mass systems demonstrated in Fig.5.

Modules are controlled according to local LQR law (a) in all cases. No connective is controlled in Case 1 and connectives are also controlled according to local LQR law (b) in Case 2. Connectives are controlled according to separation control law in Case 3 and Case 4. Design parameters are different in these two cases. Two hierarchical configurations shown in Fig. 6 are introduced in Case 5 and Case 6, respectively. In both cases, connectives between first generations (J1) are controlled according to local LQR law (b), and connectives between second generations (J2) are controlled according to separation control law (c).

These two control strategies are especially effective for large space structure systems of which we can not test total system on ground. Local LQR law is designed for small systems, however, it still needs identification of its system for Riccati equations. On contrary, separation control does not need such system identification. Then it is reasonable control strategies to apply LQR law on small systems tested on ground and to apply separation control on connectives between such small systems.

Figure 7 demonstrates simulation results. We treat vibration suppression problems using the six different control strategies shown in Table 1. 50 different external forces distributed in N(0,1.0) are assumed and Fig.7 shows range of control energy of these 50 results. Black bar, green bar and blue bar are lc=0.001, lc=0.01, and lc=0.1, respectively. Fig 7 (1) , (2), and (3) are concerning modules, connectives, and total (=modules + connectives), respectively. Results of Case 1 show that control energy in case of lc=0.1 is larger , because vibration energy of connectives can not be negligible in this case. Results of Case 2 show that control on connectives reduce control energy of both modules and total. Results on Case 3 and Case 4 show that it takes too much cost for separation control and it is not so effective. Results on Case 5 and Case 6 show that mixed control based on hierarchical configuration reduce control energy of both modules and total system. These effects are same as Case 2. However, separation control is effective for especially large space structure systems stated above. Then these results are important.

| Table 1. Six control strategies for one-dimensional spring mass model |
|----------------------------------|------------------|-----------------|-----------------|
| module                          | connective       | Hierarchical configuration |
|----------------------------------|------------------|-----------------|-----------------|
| Case 1                           |                  |                 |                 |
| Case 2                           |                  |                 |                 |
| Case 3                           |                  |                 |                 |
| (a)                              |                  |                 |                 |
| Case 4                           |                  |                 |                 |
| Case 5                           |                  |                 |                 |
| (b)                              |                  |                 |                 |
| Case 6                           |                  |                 |                 |
| (b)                              |                  |                 |                 |
Figure 7. Distributions of control Energies for fifty different external forces.

Case 1: modules (LQR), Case 2: modules (LQR) + connective (LQR)
Case 3: modules (LQR)+ connectives (separation A), Case 4: modules (LQR)+connectives (separation B)
Case 5: hierarchical configuration A, Case 6: hierarchical configuration B
C. One-dimensional model with rigid body modules and simulation results

Models with rigid body modules and bending connectives are shown in Fig. 9. A module model is hexagonal prism (1.0m in side length, 0.5m in height, and 10kg in mass). A bending connective is composed of two cylinders (0.01m in radius, 0.05m in length, and its material is steel) and one revolve joint which reveals bending property. And connection between module and connective is assumed to be rigid. Damping property is assumed to be structural damping. In this model, number of modules is 10. And boundary condition is fix-free. Two hierarchical configurations are also shown in Fig.9. And table 2 shows seven control strategies for this model.

Figure 8. Examples of system with rigid body modules and their hierarchical configurations.

<table>
<thead>
<tr>
<th>Case</th>
<th>module</th>
<th>connective</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(a)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-</td>
<td>(a)</td>
</tr>
<tr>
<td>3</td>
<td>(a)</td>
<td>(a)</td>
</tr>
<tr>
<td>4</td>
<td>(a)</td>
<td>(c1) ( \gamma_{J_1,J_2}^c = 0.1 )</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>(c2) ( \gamma_{J_1,J_2}^c = 0.3 )</td>
</tr>
<tr>
<td>6</td>
<td>(b)</td>
<td>(c1)</td>
</tr>
<tr>
<td>7</td>
<td>(c1)</td>
<td>(b)</td>
</tr>
</tbody>
</table>
Local LQR (a) is designed for 3-axis control of rigid body module, however, mathematical expression is the same as eq.(3). Also local LQR (b) and separation control (c) are designed for a revolve joint of a connective, however, their mathematical expressions are the same as eq.(4) and eq.(5), respectively.

Figure 9 (1) shows that maximum values of control torque of module slightly decrease in case 3-5. Figure 9(2) shows that introducing active connectives reduce control energy of both modules and total system. Simulation results introduced hierarchical configurations are shown in Fig.10. To control on connectives reduce control energy of both modules and total system. Maximum value of control torque of modules is also reduced. Systematical construction of control strategies based on hierarchical configurations also effective from the viewpoint of control energy.

Figure 9. Maximum control torque and total control energy of the rigid body module model. (One-dimensional connection : Case 1-5).

Figure 10. Maximum control torque and total control energy of the rigid body module model. (One dimensional connection : Case 1, 6, 7).
IV. Control Strategies of Two-Dimensional Hierarchical Modular Structures

A. Two-dimensional model with rigid body modules and simulation results.

Figure 11 is demonstrated two-dimensional model with rigid body modules shaped 6633E’ (Figure 1). A rigid body module and a connective are the same as one-dimensional model in Fig.8. Boundary condition and external forces based on geometrical symmetry are shown also in Fig.11, however, simulation results of 1/6 model are calculated. This model is 4th generation hierarchical modular structure. 2nd and 3rd generation module groups are also shown in Fig.11. J1, J2, and J3 mean connectives between first basic modules, second module groups, and third module groups, respectively.

![Image of two-dimensional model](image)

Figure 11. Two-dimensional model with rigid body modules which planer shape is 6633E’.

Seven control strategies shown in Table 3 are similar to one-dimensional Table 2. Control law (a), (b), and (c) are mathematically expressed by eq.(3), eq.(4), and eq.(5), respectively.

<table>
<thead>
<tr>
<th>Case</th>
<th>module</th>
<th>connective</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>(a)</td>
<td>-</td>
</tr>
<tr>
<td>Case 2</td>
<td>-</td>
<td>(b)</td>
</tr>
</tbody>
</table>
| Case 3 | (a) | (c1) $\gamma_{j_1,j_2}^e = 0.1$
| Case 4 | (b) | (c1)
| Case 5 | (c1) | (b)

Table 3. Seven control strategies of two-dimensional model with rigid body modules.
Maximum values of control torque and control energy are shown in Fig. 12. To control connectives reduce max value of control torque and control energy of modules. Total control energy is reduced slightly, however, Case 3 and Case 4 shows that only separation control takes too much cost.

B. Control consideration on two-dimensional hierarchical modular structures.

Figure 10 shows distributions of internal forces under centrifugal forces due to rotations with a constant angular velocity of 0.1 [rad/s]. Frame structures, whose shapes are shown in Fig. 1, are analyzed. The frame member is a solid steel bar with 1.0 cm radius and 1 m in length. The structures are assumed to be under centrifugal forces due to rotations with a constant angular velocity. These conditions are acceptable due to rotational symmetry of the proposed structures. Modules are connected with connective members, whose dimension and rigidity are both 1/10 of those of the module members. When at one point two modules come in contact with each other, one connective member is needed, and when two modules come in contact at one edge, two connective members are needed at both end of edge.

Only two distinctive distributions are illustrated here. Arrows show directions and magnitudes of internal forces. Their magnitudes are different between two types, since they are normalized by the maximum internal forces. Distribution of internal forces of 6363B shows that limited members of the structure transmit the internal forces. On the contrary, distribution of 6363D shows that many members transmit the internal forces. If we arrange controllers on members transmitted the internal forces, their arrangement in Type 6363B must be concentrated, and Type 6363D must be dispersed. Then from the viewpoint of number of controllers, Type 6363B would be selected. However, control forces required in Type 6363B are stronger than in Type 6363D. From the viewpoint of total control energy, two cases may be equivalent. This means that hierarchical modular structures can provide various control strategies for the structures having the same number of basic modules.
Figure 13. Distribution of internal forces of hierarchical modular structures under centrifugal forces.
V. Conclusion

Control strategies of hierarchical modular structures are presented. It is shown that they systematically provide various control strategies using simple and local control laws. Some simulation concerning vibration suppression of one-dimensional and two-dimensional models of hierarchical modular structures are presented. And it is shown that introducing active connectives into hierarchical modular structures are effective for realization of autonomous decentralized structural systems.

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References