RADIO PROPAGATION THROUGH THE TURBULENT INTERSTELLAR PLASMA

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1. INTRODUCTION

In recent years there has been a surge of interest among radio astronomers in the effects of wave propagation in the inhomogeneous interstellar plasma. At the time of my earlier review (Rickett 1977; hereinafter R77) the subject was of interest to observers of pulsars and to a lonely few theoreticians of plasma turbulence in the interstellar medium (ISM). The reasons for the recent interest were by no means foreseen in 1977. They derive partly from new observations, partly from theoretical advances (particularly, the recognition of refractive effects in strong scintillations), and partly from developments in VLBI. The various studies of scattering phenomena lead both to an increased awareness of the “radio seeing” conditions in the Galaxy and to an increased knowledge of the irregular (and apparently turbulent) interstellar plasma.

Wave propagation topics have figured in several recent conferences—in particular, the Beijing Union Radio Scientifique International/International Astronomical Union (URSI/IAU) seminar in May 1989 on “Radio Astronomical Seeing,” which included effects due to the neutral atmosphere, the ionosphere, the solar wind, and the interstellar medium. A 2-day workshop on “Radio Wave Scattering in the Interstellar Medium” was held in January 1988 at the University of California, San Diego (UCSD). I have drawn on the proceedings of the latter meeting.
(Cordes et al 1988a) as well as on the general literature in preparing this review. The subject has now matured to the point where a review is indeed appropriate. The main part of this paper (Section 2) reviews the observations and advances in interstellar scattering and their impact on radio astronomy, with little discussion of wave propagation theory. However, advances in propagation theory have been very important and are described more fully in Appendix A. A brief discussion of interstellar plasma turbulence is given in Section 3.

2. INTERSTELLAR SCATTERING

There has been an important interplay between scattering theory and observation in recent years. However, here I focus on the astronomical implications, and readers interested in the theoretical development should follow Appendix A in parallel.

2.1 Scattering Regimes and Source Diameters

The various regimes of radio-wave scattering and scintillation are introduced here. There are many observable quantities that are perturbed by interstellar scattering (e.g. Rickett 1988). However, it is most fruitful to divide the regimes of scattering into weak and strong according to whether the rms fractional intensity fluctuation (scintillation index) is much smaller or greater than unity, respectively, for a point source on a chosen line of sight. A typical radio observation (through more than, say, 200 pc of the galactic scattering medium and at frequencies of less than a few gigahertz) is in strong scattering. At frequencies above, say, 10 GHz the scattering is typically weak. Essentially all radio observations are very strong in the alternative sense that the rms phase is greater than 1 radian. The rms phase deviation is, however, hard to measure and to predict theoretically, since it depends on the largest scales present along a light of sight and on whether these largest structures are to be thought of as random or deterministic. Here I use the word scintillation when referring to intensity fluctuations, and the word scattering for more general perturbations of the wave field.

THE ELECTRON DENSITY SPECTRUM

The different phases of the ISM have received considerable attention in recent years (e.g. McKee & Ostriker 1977, McCray & Snow 1979). We are concerned here with the distribution of electron density, which is greatest in the “warm intercloud phase” and is less in the “hot coronal phase.” We assume that the plasma density irregularities can be characterized by a spatial power spectrum \( P_3N(\kappa) \), where \( \kappa \) is the (three-dimensional) wave number. This function is the
three-dimensional Fourier transform of the spatial correlation function of electron density deviations from the mean. It is this “electron density spectrum” that we hope to determine from measurements of scintillation and scattering. We model it as a power law (Equations A1, A2), with a strength parameter \( (C_1^2) \) that is uniform in a galactic disk, but find evidence (Section 2.4) for localized enhancements distributed randomly in the inner Galaxy.

Lovelace (1970) was the first to propose a power-law electron density spectrum. Lee & Jokipii (1976) suggested a power law over many decades of wavenumber, as for neutral turbulence, and they extrapolated from the parsec scale of interstellar clouds all the way to the “microscales” responsible for radio scattering, suggesting that the cloud size was an outer scale of a “turbulent” process with a spectral exponent \( (\beta) \) equal to that of Kolmogorov turbulence in a neutral gas. These ideas were explored further by Armstrong et al (1981). Their implications for the interstellar plasma are the subject of Section 3.

**Weak Interstellar Scintillations (WISS)** In weak scattering, a point source will show intensity scintillations that can be characterized by a scintillation index \( (\ll 1) \) and a time scale. For power-law models of the density spectrum, the spatial scale for weak intensity variations is approximately the Fresnel scale \( r_f \) (time scale \( t_f \)) (see Appendix A for the formal description). The weak scintillation spatial and temporal scales can be approximated as

\[
\begin{align*}
r_f &\equiv (L/k)^{0.5} \approx 1.2 \times 10^6 \sqrt{L_{\text{kpc}}/f_{\text{GHz}}} \text{ m}, \\
t_f &\equiv r_f/V \approx 6.7 \sqrt{L_{\text{kpc}}/f_{\text{GHz}}(50/V_{\text{km s}^{-1}})} \text{ hr}.
\end{align*}
\]

Here \( L \) is the effective distance through the interstellar scattering medium, \( k \) is the radio wave number at frequency \( f_{\text{GHz}} \), and \( V \) is the velocity of the diffraction pattern with respect to the Earth (50 km s\(^{-1}\) is typical of interstellar motion relative to the Earth, except for pulsars with larger proper motions). The point-source scintillation index for a given line of sight should increase a little faster than linearly with wavelength, until saturation near a value of unity. The reason that scintillations are not seen for all radio sources is that their angular extent is usually greater than \( r_f/L \), so that scintillation patterns from different parts of the source overlap and smear each other out, eliminating a detectable variation. This, of course, is the same reason that planets do not twinkle, whereas stars do. Thus, sources with diameters greater than about \( \theta_{\text{weak}} \) will have reduced scintillation:

\[
\begin{align*}
\theta_{\text{weak}} &\approx r_f/L \approx 8 \times 10^{-6} \sqrt{L_{\text{kpc}}/f_{\text{GHz}}} \text{ arcsec}.
\end{align*}
\]
Almost all sources exceed this diameter, and their index will be reduced and the time scale increased by the ratio of the intrinsic diameter to $\theta_{\text{weak}}$ (Rickett 1986; hereinafter R86). However, pulsars are small enough (i.e. smaller than $\theta_{\text{weak}}$) and at centimeter wavelengths show WISS (e.g. Backer 1975). It may also be that centimeter-wavelength source variations occurring over times shorter than about 1 day (Quirrenbach et al 1989a,b) are also WISS. If so, studies of these variations have the power to detect radio source diameters at the microarcsecond level.

**STRONG INTERSTELLAR SCINTILLATIONS** The "strength of scintillation" increases with wavelength $\lambda$ and with distance $L$ (i.e. the galactic pathlength). Thus, as we go from a weak scintillation condition to greater $\lambda$ or $L$, the point-source scintillation index ($m_{\text{point}}$) will approach unity. Further increases in $\lambda$ or $L$ will cause $m_{\text{point}}$ to increase above unity, saturate, and then decrease asymptotically toward unity. However, there is an important change in the spatial characteristics as the scintillation becomes strong. The diffraction pattern takes on a two-scale character, by which is meant that the spatial power spectrum of the intensity pattern exhibits two regimes. Such spectra are sketched in Figure 1. The higher wave numbers are due to diffraction, while the bump at lower wave number is due to refraction [see Spangler (1988) for a review of diffractive interstellar scintillations (DISS)]. If these regimes are characterized by scales $s_d$ and $s_r$, respectively, it follows that $s_d s_r \approx r_I^2$. Further into strong scattering $s_d$ decreases, and so $s_r$ increases. A physically intuitive understanding of these two components comes from the concepts of the scattering angle and the scattering disk.

We first define the field coherence scale $s_0$ as the lateral separation, at an observing plane, across which there is a 1-radian rms difference in the phase calculated along a straight-line path from the source to the observer. From this, the effective scattering angle $\theta_s$ can be defined:

$$\theta_s = 1/(k s_0).$$

At an observing location, the summing and interference of waves arriving from the angular spectrum (of width $\theta_s$) cause the diffractive amplitude variations. A lateral displacement of $s_0$ substantially changes the relative phases of these interfering waves and, thus, the amplitude. Hence, the diffractive amplitude scale is nearly equal to the field coherence scale ($s_0 \approx s_0$). It is useful to define a strength-of-scattering parameter $u$ (strong scattering is $u > 1$):

$$u = r_I/s_0.$$

The concept of the scattering disk is also useful; it is the largest transverse
Figure 1  The power spectrum of intensity fluctuations for a screen (with Kolmogorov spectrum) plotted logarithmically versus the magnitude of the two-dimensional wave number, which is normalized to $r_t^{-1}$. The curves have the proper slopes and widths, and, though not precise, they show how the spectral width increases with the strength of scattering ($u$), which is marked against each curve. For $u > 1$, the peak at low wave numbers ($u^{-1}$) is due to refraction, and the plateau, which extends to high wave numbers ($u$), is due to diffraction. The variance, given by a two-dimensional integration over wave number, has contributions from both components. The diffractive variance is unity independent of $u$, so the plateau level decreases as $u^{-2}$. The refractive variance decreases only slowly with $u$. In weak scintillation ($u < 1$) there is a single peak at $r_t^{-1}$, and the variance and peak level vary as $u^{5/3}$.

A scale that influences the signal arriving at a single observing point. At a typical distance to the scattering medium, its radius is $L\theta_s$. In an extended scattering medium, we should think of $L\theta_s$ as the typical cross-sectional radius of a scattering volume, which in fact will be cigar shaped. Since the medium causes increasing phase perturbations with scale, there are large phase differences across the scattering disk. These cause partial focusing or defocusing, giving “refractive” amplitude variations. A lateral displacement of the observer by $L\theta_s$ corresponds to a new scattering volume, and so to a change in the refractive modulation. Thus, the refractive scale $s_r$ is approximately equal to the radius of the scattering disk:

$$s_r \approx L\theta_s = ur_t,$$

Using Equation 2.4, we quickly obtain the result $s_0s_r \approx r_t^2$. The refractive scale is a factor of $u$ greater, and the diffractive scale a factor of $u$ smaller, than $r_t$. In a medium with a “steep” spectrum that is enhanced at large
scales, the above discussion avoids some of the subtleties. However, it remains accurate provided the scattering angle is defined in terms of an instantaneous angular spectrum, as opposed to an ensemble average angular spectrum, which includes deviations in angle of arrival over extremely long scales that do not modulate the amplitude. This point is discussed in Appendix A and in Section 2.5 under “Image Wander.”

For an ideal point source the total variance in intensity can be greater than 1, particularly near $\mu \approx 1$. In the limit of strong scattering the refractive variations can be modeled as multiplying the diffractive variations. Then a modulation index $m_d$ and $m_r$ can be defined for a total variance:

$$m_{\text{point}}^2 \approx m_d^2 + m_r^2 + m_d^2 m_r^2.$$  \hfill (2.7)

The diffractive interstellar scintillations (DISS) have $m_d^2 = 1$. Typically, observed refractive interstellar scintillations (RISS) have $m_r^2$ of about 0.1, decreasing slowly to zero as $\mu$ increases.

The influence of source diameter can be considered in the same fashion as for weak scintillation. A source must be smaller than the angular sizes $s_d/L$ and $s_r/L$, respectively, to show diffractive and refractive scintillation. Since $s_d$ is smaller than $r_d$, sources showing DISS have to be even smaller than is necessary for WISS. Dennison & Condon (1981) observed a sample of 22 sources that were suspected of being extremely compact on the basis of low-frequency variability. They found none to exhibit DISS and so set lower limits on their angular diameters in the range of microarcseconds. Their conclusions were in agreement with earlier searches for DISS in nonpulsar sources (Condon & Backer 1975, Armstrong et al. 1977, Condon & Dennison 1978).

However, the limit for RISS is less stringent; using Equation 2.6, we see that if the intrinsic angular size $\theta_{\text{source}} < \theta_s$, then RISS will be observable at a level characterized by $m_r$. This limit is of the order of milliarcseconds at 1 GHz and $L \approx 0.5$ kpc. Such refractive variations are discussed in the following sections. The refractive regime was overlooked in the radio literature until Rickett et al. (1984; hereinafter RBC) proposed it as an explanation for slow variations in pulsars and in other classes of radio sources. Several authors have now studied a wide variety of refractive effects [e.g. Cordes et al. 1986 (hereinafter CPL), Romani et al. 1986 (hereinafter RNB)]. It is interesting to note that even though the theory of the two regimes was already developed in the optical literature (e.g. Prokhorov et al. 1975), we overlooked the application to interstellar scattering until Sieber (1982) published his study of slow-pulsar amplitude variations. Our wave propagation group at UCSD had even discussed the theory (e.g. Rumsey 1976) and observed the two regimes in optical measurements (Coles & Frehlich 1982).
2.2 RISS of Pulsars

Sieber’s (1982) recognition that the slow amplitude variations of pulsars are due to interstellar propagation was the essential first step leading to the identification of RISS. Refractive effects from discrete structures in the interstellar medium had been proposed earlier as an explanation of the problematical low-frequency variability of some radio sources (Shapirovskaia 1978), but Sieber's work led RCB to propose refractive scintillation from the same “turbulence” spectrum that causes diffractive scintillation. Most of the data analyzed by Sieber (1982) were from early pulsar investigations (Cole et al. 1970, Rankin et al. 1974, Helfand et al. 1977). Sieber noticed a marked increase in time scale with dispersion measure and hence distance. Clearly this flagged the variations as extrinsic rather than intrinsic. As he noted, the sense of the relation is exactly opposite to that known for DISS, in which \( s_0 \) decreases with distance. However, it is in good agreement with Equation 2.6 for RISS. An important corollary was also recognized by RCB: namely, that RISS might account for the low-frequency radio-source variations that had lacked a reasonable intrinsic interpretation. Dennison & Condon (1981) had also noted that a propagation process could explain the low-frequency variables.

The depth of the slow pulsar amplitude variation was, however, shown to be greater than that expected for RISS \( (m_t) \) in the standard Kolmogorov model. This led several investigators to question the Kolmogorov model. Most notably, Blandford & Narayan (1985) proposed that the plasma density power spectrum for the ISM was a power-law function with an exponent \( \beta \geq 4 \). This proposal was driven partly by concern for the theoretical difficulties in supporting a turbulent cascade in the interstellar plasma. The Kolmogorov and related power-law models have attracted interest because of the analogy that can be made with neutral gas turbulence; by contrast, the steep spectra do not need a turbulent explanation. Blandford & Narayan and coworkers have since explored their proposal in detail. These “steep-spectrum” models are strongly refracting, and the scattering analysis thus requires extra care. The shape for the density spectrum is still debated, and its determination continues to be a central part of ISS investigations [see Narayan (1988) for a summary of how the various observations constrain our knowledge of it]. He emphasizes the need to solve the inverse problem of determining the density spectrum from the observations. Unfortunately, the problem is neither linear nor invertible, and thus one must resort to model fitting to the data, a process that by no means has a unique solution.

Following Sieber’s paper, there have been some new RISS observations (Stinebring & Condon 1990, Rickett & Lyne 1990). Figure 2, reproduced
Figure 2  Refractive scintillation index versus $\delta v_d / \nu$, which varies inversely with $u$. For a Kolmogorov spectrum, we have $\delta v_d / \nu \sim 2u^{-2}$. The observational estimates are plotted with their error bars [see Rickett & Lyne (1990) for detailed references]. The lines give the first-order theory for a uniform medium/Kolmogorov spectrum and for inner scales of (a) 0, (b) $10^7$ m, (c) $10^8$ m, (d) $10^9$ m, and (e) $10^{10}$ m. The observations are all above line (a) and substantially below unity, which is predicted for steep-spectrum models; they are thus consistent with an inner scale of $10^8$–$10^9$ m.

from the latter paper, summarizes our state of knowledge on $m_r$. In brief, the values, though poorly estimated, are definitely higher than the predictions of the Kolmogorov model but are significantly below the 100% predicted for the steep power-law models. The dashed lines in Figure 2 show approximate predictions for a Kolmogorov density spectrum with a high-wave-number cutoff due to an inner scale (Coles et al 1987). There is no consensus, although the model of Shapirovskaya & Sieber (1984) now seems unlikely, and more observations of pulsar RISS would be helpful.

2.3 RISS of Other Radio Sources

Interstellar scintillation is well established for pulsars; this means that similar effects must be occurring for all radio sources. Though there appear
to be no cases in which sources other than pulsars have small enough angular diameters to exhibit DISS, refractive variability of other galactic and extragalactic sources can be important. In some cases RISS may explain all of the observed variations and remove the need for the sometimes exotic source models invoked to explain the compactness of objects based on light travel-time arguments. The various categories of variable sources were considered by R86 in terms of a galactic disk distribution of scattering and are only briefly reviewed here.

LOW-FREQUENCY VARIABLE SOURCES Sources with 3–30% variations over periods of months observed below 1 GHz have been labeled as low-frequency variables. The high brightness temperatures implied by such intrinsic variations were a serious enough astrophysical dilemma to be the topic of a National Radio Astronomy Observatory (NRAO) workshop (Cotton & Spangler 1982). It is now clear that RISS is at least partly responsible for the variations (R86, Shapirovskaya 1985).

Spangler et al (1990) have statistically analyzed the long data set observed at 408 MHz at Bologna with a view toward examining a RISS explanation. They found difficulties with a completely RISS explanation and concluded that there were often two characteristic time scales (for 20 out of 39 sources studied), suggesting that the longer of them was intrinsic and the shorter was due to RISS. They did not find the close relations between time scale, modulation index, and galactic latitude, expected in the simplified RISS model of R86. However, their data definitely show that RISS contributes to the variations, in accord with Cawthorne & Rickett (1985) and Gregorini et al (1986), who found an increasing fraction of low-frequency variables at low latitudes.

Refinements in the analysis and in the RISS model are needed. Mantovani et al (1990) obtained better agreement with the RISS model for a subset of 24 Bologna sources with VLBI data at nearby frequencies. They corrected the index values of Spangler et al (1990) for extended (nonscintillating) components and found an average latitude dependence in accord with the disk distribution of scattering, but with an angular broadening at the galactic pole that was a factor of ~3 less than the nominal value. The uniform disk distribution is probably an oversimplification, needing random enhancements in scattering along the line of sight (Section 2.4). The conclusion from these analyses is that there are both RISS and intrinsic variations at 408 MHz. The latter are slower and typically below the $10^{12}$-K limit, and so the chief puzzle associated with low-frequency variables is removed. However, Slee & Siegman (1988) have reported an analysis of 412 sources observed with the Culgoora array at 80 and 160 MHz in which they found that nearly half of their sources were
10–30% variable over times of 1 month or 1 yr. However, the expected latitude dependences were barely significant, and thus they concluded that normal RISS is not a satisfactory explanation.

**GALACTIC PLANE VARIABLES** Sources at low galactic latitudes have a greater chance of enhanced scattering, and so a greater fraction may show RISS with time scales of tens of days at centimeter wavelengths. However, the random distribution of enhanced scattering precludes detailed predictions. Dennison et al (1987) posed RISS as the cause of variations in the low-latitude sources 2013 +370 and 0355 +208, observed at 2.7 and 8.3 GHz during 1979–86. Hjellming & Narayan (1986) studied the inverted spectrum variable source 1741 –038 over a year with time resolution down to a few days at frequencies of 1.49, 4.9, 15, and 22 GHz. Their results showed a 300-day intrinsic variation at 22 GHz, a largely RISS 20-day variation at 1.49 GHz, and variation at 4.9 GHz in which both effects were important. These findings are in agreement with the measured angular broadening and imply a level of scattering 100 times stronger than for the simple galactic disk model for a line of sight at 13° galactic latitude. The 20-day time scale at 1.49 GHz implies a distance of about 500 pc for the scattering—considerably closer than for the disk model but possibly compatible with a clumpily distributed enhanced scattering (see Section 2.4).

Gregory & Taylor (1986) have made careful surveys of the galactic plane for variable radio sources at frequencies of 1.4 and 5 GHz. They classify their variable sources as short-term (1–20 days) or long-term variables (∼1 yr). Many of these sources may be extragalactic, and so the distance through the galactic plane may be quite large and strong scattering conditions are likely to apply even at 5 GHz. However, RISS was not considered as part of the interpretations by Gregory (1987), Duric & Gregory (1988), and Duric et al (1987); rather, they concluded that they were observing intrinsic variations from a variety of galactic and extragalactic sources. Similarly, Duric et al (1989) observed strong variations in the cores of two extended double radio sources at both 1.45 GHz and 4.87 GHz, which they interpreted as intrinsic. Though RISS complicates the interpretation, sources as compact as is implied by an intrinsic variability must also exhibit RISS.

**RAPID VARIATIONS (Flickering)** In a phenomenon that he called flickering, Heeschen (1984) found variations of a few percent at 3.3 GHz over times of about 10 days for a large fraction of flat-spectrum radio sources. As for low-frequency variables, intrinsic flickering would imply brightness temperatures well above $10^{12}$ K. Although a RISS explanation was rejected in the two-frequency study by Simonetti et al (1985), it was supported by
Heeschen & Rickett (1987), who found an increasing level of flicker with decreasing galactic latitude. Once again we have the problem of determining whether the variations are intrinsic, extrinsic (RISS), or a mixture.

Even faster variability has now been reported in other high-latitude sources. Heeschen et al. (1987) observed 15 compact sources at 2.7 GHz at 2–4 hr intervals for 3–4 days at three epochs and detected variations of a few percent, which they classified in two types. The first they identified as RISS, and the second as either intrinsic variations or variations due to refraction from discrete regions of the ISM. Dreher et al. (1986) observed a 1.8% decline in flux over 7 hr and a 0.5% variation in 15 min in the 5-GHz flux of OJ 287; these variations could also be RISS, although Dreher et al. posed an intrinsic explanation. Quirrenbach et al. (1989a, b) observed rapid variations from 0917 + 624, with peak-to-peak variations of 20–30% at both 2.7 GHz and 5 GHz, with partial correlation. The time scales were in the range 0.5–2 days, with the variations noticeably faster at 5 GHz than at 2.7 GHz. If intrinsic, the variations imply brightness temperatures of $10^{17}$–$10^{19}$ K. Quirrenbach et al. also concluded that the simple slab model of RISS required an unrealistically close distance for the scattering. However, in applying the same RISS model, I disagree, finding a time scale of 18 hr at 2.7 GHz, which is compatible with a source diameter of 0.08 mas (milliarcseconds), a velocity of 50 km s$^{-1}$, and a 500-pc distance for the scattering. The associated scintillation index would be about 30%, which could be reduced to the observed 7% (23% + 3) by an extended nonscintillating component. At 5 GHz, the time scale would require a still smaller diameter. At 2.7 GHz, it requires 0.09 Jy in, say, a 80-μas (microarcsecond) component, implying a brightness temperature of about $3 \times 10^{12}$ K, which is only slightly above the inverse Compton limit. The later observations of Quirrenbach et al. (1989b) showed remarkable variations of the linearly polarized flux by a factor of 3, which were anticorrelated with the total flux. Refractive interstellar scintillation does not cause significant variability of polarization, and thus a RISS explanation appears to be ruled out. However, as the authors point out, the superposition of orthogonally polarized core and jet emission could give the 1–2% average polarization, and RISS modulation of the compact core could give anticorrelated variations of total and polarized flux. VLBI polarimetry could test this possibility.

In generalizing, R86 considered sources with a brightness of $T_{12} \times 10^{12}$ K (i.e. a diameter proportional to wavelength). Under strong scattering conditions, RISS at latitudes $b \geq 10^\circ$ is constrained by a rate of change of flux:

$$dS/dt \lesssim S_{\nu,0}/r \sim 30T_{12}^2(\sin b)^{0.5} \text{ mJy day}^{-1}. $$
This relation shows whether an RISS interpretation requires a diameter that infringes the Compton limit.

In spite of questions, the combination of RISS and the idea that component sizes decrease with wavelength explains an important property common to many of the observations. As the frequency is increased, the high-latitude sources vary with smaller amplitudes and shorter time scales. Figure 1 of R86 illustrates this for a latitude of 45°. There is, however, another scenario that should be explored—weak interstellar scintillation. As the frequency is increased, the strength of scattering decreases, and above \( \sim 10 \) GHz it will become weak. Given the uncertainties in the model, the frequency of this transition is poorly known. In WISS the time scale is given by Equation 2.2 for a plane wave source and is lengthened by the ratio of the intrinsic source diameter to \( \theta_{\text{weak}} \) (given in Equation 2.3); this same factor governs the decrease of scintillation index over a point source, which by postulate is already less than 1.0. The WISS behavior consistent with the slab model would apply above a frequency of \( \sim 10 \) GHz (Equation 2 in R86). However, for a density spectrum with an inner scale, WISS becomes a more important possibility for the high-latitude variations discussed here. It may also be important for flare stars and might even contribute to the frequency structure observed in recent flare star studies (Bastian & Bookbinder 1987, Jackson et al 1987).

OH AND H\(_2\)O MASER SOURCES The importance of angular broadening in maser sources has been recognized for some time (see Reid & Moran 1981). This in turn indicates intrinsic diameters smaller than the scattered diameter, and so to the expectation of RISS variability as a point source. Since the sources are typically at low galactic latitudes, they are likely to be influenced by the clumpy enhanced scattering, making it difficult to predict quantitatively. Typical angular diameters at 1.6 GHz (OH) are a few mas and are smaller by roughly the square of the frequency ratio at 22 GHz; corresponding time scales depend on \( L \) and \( V \), but they will be many years for 1.6 GHz and tens of days for 22 GHz. The latter time scale could thus be studied readily, and there are indeed a number of reports of short-term variations of H\(_2\)O masers (e.g. White & Macdonald 1979, Rowland & Cohen 1986). The masers often have many features at differing velocities and positions. Whether the RISS modulation should be independent from feature to feature depends on whether the features are separated by transverse distances bigger than the scattering disk. Once again, we need to consider both intrinsic and RISS variations.

DISCRETE PROPAGATION EVENTS A remarkable new type of interstellar variability has been discovered by Fiedler et al (1987). Figure 3 reproduces their observations of source 0945+658 from 1980.5 to 1981.7. This source
Figure 3  Intensity series at 2.7 and 8.1 GHz for the discrete propagation event on 0945 + 658 observed by Fiedler et al (1987). Note the pronounced dip in the intensity, preceded and followed by local maxima that are several times the average source flux.
was observed daily at Green Bank as part of the Naval Research Laboratory monitoring program. The 2.7-GHz flux shows a notable \( \sim 50\% \) dip lasting for about 60 days, preceded and followed by maxima of \( \sim 30\% \); during the dip, the 8.1-GHz flux is also reduced but has superimposed four spikes that are as much as twice the average flux and that last for a few days. The NRL observers argue persuasively that this is a propagation effect and propose a particular explanation in terms of a discrete scattering object passing between us and the source. On approach, we receive the extra flux scattered out of the direct path. This event was the most spectacular, but they reported three events in 395 days of observing 160 sources. In a reanalysis of all the Green Bank data, Fiedler et al (1988) reported a total of 8 comparable events, although for the other 7 events the variation was only strong at 2.7 GHz. These events were named by Fiedler et al as “extreme scattering events,” in accord with their model outlined above.

Others have referred to them as “enhanced refraction events,” following Romani et al (1988), who proposed an explanation in terms of discrete refracting structures; Romani et al emphasized the possibility that the structures could be filamentary or sheetlike rather than spherical. Romani (1988) and Clegg et al (1988) have concentrated on refraction from possible large-scale ionized structures, particularly interstellar shocks, which seems a likely geometry. Romani et al also point out that the same refraction could cause the “double-imaging” episodes in the dynamic spectra of pulsars (Section 2.5).

A major unresolved question is whether the structures responsible are related to the random density irregularities that cause ISS or whether they are discrete, unrelated ionized regions. As discussed below, we already must consider the scattering medium to be very clumpy, and it would be attractive to include these new structures as particularly dense clumps. We also need to know how these structures fit into our physical picture of the different parts of the ISM.

### 2.4 Distribution of Galactic Scattering Material

The discussion so far has emphasized questions of distribution of electron density versus wave number, i.e. the spatial spectrum. Our zero-order model has been a power-law dependence of spectral density on wave number, independent of position in a uniform galactic disk, taken to be about 1000 pc thick. This was the model studied in R77; the conclusion there was that, whereas the spectral shape could well be of the Kolmogorov form, the spatial distribution was not uniform, tending to stronger scattering density with increasing galactic pathlength.

A major study of the galactic distribution of scattering material was completed by Cordes et al (1985; hereinafter CWB). They observed the
diffractive scintillation of 31 pulsars, and for each they estimated the characteristic frequency width $\delta v_\text{d}$ of the scintillations. They also assembled observations from as many frequencies as possible on 5 pulsars and demonstrated that the frequency scaling laws for $\delta v_\text{d}$ were close to those expected for the Kolmogorov spectrum. They then combined these observations with other measurements from the literature and investigated their dependence on galactic coordinates. In strong scattering, the diffractive scintillations become very narrowband (see Section 2.6), varying randomly on frequency separations $\delta v_\text{d}$ and on a time scale $t_\text{d} (\approx s_0/V)$. The measurements of $\delta v_\text{d}$ can be used to estimate the average scattering parameter ($C_N^2$, defined in Equation A1) under an assumption of a uniform distribution of scattering electrons along the line of sight to each pulsar (see CWB for detailed formulas). Their results showed a very wide range in the derived values of $C_N^2$, in disagreement with the assumption of uniformity. They showed that the variation of $C_N^2$ with galactic coordinates is compatible with a two-part distribution of scattering material: A uniform disk with a scale height of $\pm 500$ pc and $C_N^2 \approx 10^{-3.5} \text{ m}^{-6.67}$ and a superimposed clumpy distribution with a scale height of 100 pc and $C_N^2$ in the range $10^{-3} - 1 \text{ m}^{-6.67}$. The clumpy material has a filling factor in the range $10^{-3} - 10^{-4}$ and a mean free path of 1–10 kpc for a line of sight to intersect a region of enhanced scattering. These parameters relied on assuming a scale of about 1 pc for the clumps. Note that if a 5-kpc line of sight has an increase of $10^3$ in distance-averaged $C_N^2$, the local $C_N^2$ in the clump would have to be increased by the ratio of path length to clump size (i.e. a further $5 \times 10^3$), making it $10^6$ times the background. It becomes of great interest to identify what galactic objects or phases of the medium cause such enormously enhanced scattering.

The CWB model of the scattering distribution has been the starting point of several further studies. Rao & Ananthakrishnan (1984) used the method of interplanetary scintillations to demonstrate the presence of enhanced scattering for lines of sight within about 5° of the galactic center. Dennison et al (1984) surveyed 29 low-latitude ($< 5°$) compact extragalactic sources using VLBI at 408 MHz. They found that many of the sources with longitudes close to the galactic center were resolved. This they interpreted as enhanced angular broadening due to the same distribution of irregularities. Their combined results were summarized by Cordes et al (1984). Alurkar et al (1986) reported temporal broadening of 33 pulsars at frequencies of 410, 160, and 80 MHz. They combined these measurements with other published scattering observations and proposed a single distribution of density inhomogeneities that is highly clumped and has a scale height at least as great as that of the pulsars. Their proposed distribution is strongly peaked near the galactic center, with a fall-off by
a factor of 1000 as the galactocentric distance increases from 1 to 10 kpc. Thus, they suggest that the diffuse uniform component proposed by CWB is not separate but is part of a single clumpy distribution with a large scale height (>0.5 kpc). They confirmed the wide range of scattering on sources within a few degrees.

Figures 4a and 4b are from Cordes et al (1988b), who recently summarized our knowledge of this subject and reaffirmed the two distributions. For each object there is an estimate of distance $L$, mostly derived from pulsar dispersion measures using the electron distribution model of Lyne et al (1985). In Figure 4a, $C_{N}^{2}$ from high latitudes ranges from $10^{-4}$ to $10^{-2}$ m$^{-6.67}$ with a mean near $10^{-3.5}$ m$^{-6.67}$. Cordes et al concluded that the high-latitude objects are consistent with an exponential $z$-distribution with scale height 0.5–1 kpc and a dependence on galactocentric distance, which they model as a Gaussian with a 1/e radius of 7 ± 2 kpc. Low latitudes show an even larger range ($C_{N}^{2} \sim 10^{-4}$–$10^{0.5}$ m$^{-2.0/3}$) and a higher mean. However, since the low-latitude objects do not sample as great a range of heights as the higher latitude (and nearer) objects, the extremely variable strength of $C_{N}^{2}$ only weakly constrains the scale height of the enhanced scattering. Attempts to find the galactic objects or regions responsible for enhanced scattering remain inconclusive (see Section 3). Meanwhile, evidence for specific, very strongly scattered lines of sight continues to accumulate.

2.5 Angular Broadening

**Theory** In principle, angular broadening provides one of the most direct ways to measure scattering in the ISM; of course, the conditions for such measurements to be successful are just those where scattering causes the worst interstellar “seeing.” The synthesis of an image in radio astronomy usually relies on correlations from multiple interferometer baselines. A single interferometer measurement (on baseline $\sigma$) provides an estimate of the average visibility $E(s)E^*(s+\sigma)$ (see Appendix A). Equation A3 shows that (given sufficient integration) the visibility leads to a direct estimate of the structure function for geometric phase $\phi$. However, if there is no averaging in time, space, or frequency, the visibility will be highly variable—the analog of speckle in the image of a filled-aperture telescope. Thus, in a synthesis observation, the applicability of Equation A3 depends on receiver bandwidth, coherent integration time, intrinsic source structure, and method used for calibration (self-calibration or otherwise).

Assuming the conditions for strong scattering, there are independent fluctuations of visibility over a distance $s_0$ and frequency interval equal to that for intensity scintillations ($\delta v_d$ in Equation A14). A typical observing bandwidth $B$ may include many such independent “scintles,” reducing the
Figure 4  Plots (from Cordes et al 1988b) of the line-of-sight-averaged $C_N^2$ values versus (a) distance from the Earth and (b) galactic coordinates. Plot (a) shows the erratic and increasing scattering at low latitudes and great distances, and (b) demonstrates that these increases are strongest toward the inner part of the galactic disk. The origin of the localized, strongly enhanced "turbulence" is actively being sought.
visibility variations. Similarly, the coherent integration time $T$ may be longer than the time $t_d \sim s_0/V$ for a scintle to cross. And, finally, if the source is extended sufficiently to smear the pattern spatially ($\theta_{\text{source}} > s_0/L$) but too small to be resolved on the baseline, there will be further averaging. These ideas were discussed qualitatively in an important paper by Cohen & Cronyn (1974). In practice, source smearing is likely to dominate typical VLBI observations (Rickett & Coles 1988). Narayan et al (1989) discuss the possibility of estimating an image with very high angular resolution ($s_0/L$); the same resolution is achievable from intensity scintillation, but it only estimates the squared visibility.

A quantitative analysis in weak scattering was presented by Cronyn (1972), but it has been little used. In optical observations atmospheric twinkling is a well-studied phenomenon, and techniques (e.g. speckle interferometry) for avoiding its limitations have been developed [see Coulman (1985) for a review]. A careful quantitative analysis of the variations in a single visibility measurement in strong scattering has recently been made by Goodman & Narayan (1989) and Narayan & Goodman (1989). They classified the averaging regimes as “snapshot,” with 100% variation due to diffractive speckles; “average,” with partially smoothed speckle and full refractive variability; and “ensemble average,” which also smoothes refractive variability and for which Equation A3 applies. They did not address the effects of self-calibration.

There have been several recent VLBI observations of angular broadening in the ISM. Results have been presented in plots of angular diameter versus wavelength or as a scattered image (restored with the usual synthesis techniques) or as the magnitude of visibility versus baseline. The latter form allows a direct comparison with Equation A3, from which one can estimate the phase structure function $D_\phi$. A particular point of interest is to estimate the exponent $\alpha(=\beta-2)$. The inner-scale density spectrum (Coles et al 1987) implies $\alpha = 2$ for baselines shorter than their proposed inner scale near $10^9$ m. Many of the observations discussed below give results that are consistent with the Kolmogorov value of $\alpha = 5/3$, in disagreement with this inner scale. However, the interpretation of the data is still a matter of debate. The first problem, as discussed by Goodman & Narayan (1989), is a bias at low visibility. The observations are in the “average” regime, and if the magnitude of the visibility is estimated, there is an upward bias that decreases as the inverse square root of the number of independent scintles in time $T$ and bandwidth $B$. At low visibility this is the same bias that occurs in estimating the magnitude of the correlation coefficient for any pair of uncorrelated random quantities. Such a bias reduces the apparent exponent $\alpha$ but typically has been ignored. A second problem is the effect of intrinsic source structure; the intrinsic source
visibility multiplies the scattered visibility, steepening the decorrelation versus baseline—a bias in the opposite direction. The third problem is the influence of self-calibration, which essentially centers the image, removing a linear phase gradient from the field. Since a $D_\phi$ exponent of 2 corresponds to a linear phase gradient, self-calibration may cause a bias, but no proper study has been made. The VLBI results for the exponent $\alpha$ should thus be viewed with caution. Note, however, that Spangler & Gwinn (1990) conclude that the published values demonstrate an inner scale of $10^6$ m.

**OBSERVATIONS** In recent years several groups have applied the powerful angular resolution of VLBI to interstellar broadening, particularly at meter wavelengths. The criterion for sufficient resolution is that the baseline is greater than $s_0$, which is more readily satisfied at larger wavelengths, since $s_0$ decreases as $\lambda^{-1.2}$. Conversely, the conditions under which VLBI will be uncorrupted by ISS effects favor shorter wavelengths and baselines. Many heavily scattered objects have been found, which confirms the ideas of Cordes et al (1984) that there is low-latitude enhanced, clumped scattering.

VLBI observers have found a number of radio sources whose angular extent vary approximately as the wavelength squared, a pattern characteristic of plasma scattering. The low-latitude survey at 408 MHz by Dennison et al (1984) revealed strong but variable levels of scattering on nearby lines of sight. Fey et al (1988) studied the angular broadening in the Cygnus region of the Galaxy, finding similarly large variations over only a few degrees. Both groups noted the large scattered diameter of $2048 + 313$ (CL 4) which lies at 8° latitude behind the Cygnus Loop. Spangler et al (1986) measured the angular scattering in the vicinity of supernovaremnants. Whereas they found several heavily scattered sources, they were unable to establish the supernova association. Spangler & Cordes (1988) further investigated the question for the remnant G33.6 + 0.1. Using the VLA at 333 MHz, they observed an anisotropic (3 : 1) and very heavily scattered image for $1849 + 005$. They also observed 45 other nearby sources at 1.46 GHz and concluded that the scattering increased between 42° and 22° longitude, and rejected the supernova (G33.6 + 0.1) as the cause. Presumably, the heavy scattering of pulsar PSR $1849 + 00$ (Clifton 1986) has the same cause. The elliptical image of $1849 + 005$ is the strongest evidence for anisotropic scattering; such a large value implies a truly anisotropic density structure, rather than the smaller randomly changing anisotropy predicted by RNB due to differing degrees of phase curvature in orthogonal directions. If verified, the large anisotropy will provide important evidence for interstellar turbulence elongated by local magnetic fields (e.g. Higdon 1986; see Section 3). Narayan & Hubbard (1988) should be consulted for details of propagation theory with substantial anisotropy.
Wilkinson et al (1988) measured the strongly scattered image of Cyg X3 during its 1987 outburst. They found a 2.8-arcsec diameter at 408 MHz, in close agreement with a 1972 measurement. In a useful clue to the cause of enhanced scattering, Moran et al (1990) observed very heavy scattering for NGC 6334B, which they suggest is due to one of the lobes of NGC 6334A—a bipolar H II region.

Mutel & Lestrade (1988, 1989) used sensitive Mark III VLBI recordings at 5 GHz to study the heavily scattered source 2005 +4 03. They found a visibility decreasing with baseline, as expected, but they also found that it saturated at about 1% beyond 5000 km. Although Mutel & Lestrade initially attempted an interpretation using Equation A3, their 1989 analysis suggested that the saturation is due to refractive bias in visibilities (Goodman & Narayan 1989). Alternatively, the cause may be the bias due to insufficient averaging of the diffractive speckle, as discussed above for estimates of visibility magnitude; I estimate this to be a significant bias (∼0.6%), but it needs further details of the processing.

Backer (1988) reviewed the VLBI observations of the compact source Sag A* at the galactic center. The $\lambda^2$ scaling of its angular diameter shows clearly that it is dominated by a very heavy scattering. He noted an apparent exponent steeper than 2 for the structure function, estimated via Equation A3 for the data of Lo et al (1985). However, Lo et al did not properly account for the slight anisotropy in the visibility function. Armstrong et al (1990) dealt with this problem for the highly anisotropic broadening (10 : 1) in the inner solar wind by plotting visibility as a function of the quadratic combination of baseline components that characterizes the fitted elliptical correlations. The location of Sag A* makes it an intriguing object, but scattering hinders its study. Backer concluded that it is intrinsically very compact and variable.

Interstellar scattering is evident in distant maser sources. This causes a number of OH/H$_2$O masers to show diameters in a $\lambda^2$ ratio between 18 and 1.35 cm (Reid & Moran 1981). Diamond et al (1988) observed the diameters of 12 OH maser sources (at 1665 MHz) to increase with distance (to about 50 mas at 10 kpc). This increase is similar to the extra broadening of low-latitude extragalactic sources.

Gwinn et al (1988) reported VLBI observations of PSR 1933 +16 at 326 and 608 MHz. They found a moderate level of scattering, measuring a 50% visibility on a baseline of about $10^7$ wavelengths at 326 MHz. Their result can be combined with the temporal pulse-broadening to estimate the distance for an equivalent scattering screen—about 75% of the pulsar distance, which is a reasonable result.

Scattering can cause a (slowly variable) shift in apparent
source position, in addition to broadening it. The magnitude of such a position wander \( \theta_r \) compared with the scattering angle \( \theta_s \) is a potential discriminator for the interstellar density spectrum. In particular, the "steep" power-law models predict strong refractive shifts. The refractive shift is determined by the gradient in phase, averaged over the dimensions of the scattering disk, \( s_r = L \theta_s \). Thus, it is given by

\[
\kappa \theta_r \approx D_\phi(s_r)^{0.5}/s_r,
\]

which (if \( 0 < \alpha < 2 \)) can be expressed as

\[
\theta_r \approx \theta_s [s_0/s_r]^{1/2} = \theta_s u^{\alpha/2}.
\]

For \( \alpha = 5/3 \), this gives \( \theta_r \approx \theta_s u^{-1/3} \). In strong scattering (\( u > 1 \)), this shows that \( \theta_r < \theta_s \). In contrast, for steep spectra (\( 2 < \alpha < 4 \)) \( \theta_r \gg \theta_s \), and \( \theta_r \) depends on the outer scale (see RNB, CPL).

Unfortunately, absolute position measurements are difficult to make. The apparent separation of two sources (of true separation \( \Delta \theta \)) is also subject to an error \( \delta \theta \) due to a relative refractive shift. Since relative positions can be measured more accurately, repeated observations should reveal how \( \delta \theta \) varies with time and frequency. For a baseline \( v \), \( \delta \theta \) depends on the second difference of the phase front, which is characterized by \( F(v, L \Delta \theta) \) in Equation A12. This function also controls the development of intensity scintillations. Thus, \( \delta \theta \) becomes a less valuable discriminant between spectral models, since a square-law term in the phase structure function contributes nothing to \( F \), and it is the square-law term that is dominant for the steep spectra (see Goodman & Narayan 1985, Rickett & Coles 1988).

Gwinn et al (1988) made an interesting VLBI study of refractive shifts in a cluster of H_2O masers in Sgr B2. They determined that over the course of 6 months, the rms wander of the individual maser "spots" was smaller than 18 \( \mu \)as. This is substantially smaller than the measured angular broadening of 300 \( \mu \)as for Sgr B2. They used the theories of RNB and CPL and corrected for differential position errors, concluding that \( \beta < 3.67 \) on spatial scales up to \( 10^{11} \) m. They also reported similar preliminary results from W49. With a view to determining whether Sgr A* is truly at the galactic center, Backer (1988) made proper-motion measurements and estimated that any refractive position error was 10 times smaller than the broadened image. Mutel & Hodges (1986) and Mutel & Lestrade (1989) observed the compact extragalactic double source 2050 + 364, whose components are broadened as \( \lambda^2 \). They found a refractive error \( \delta \theta < 1 \) mas at 610 MHz, which again implies that \( \theta_r < \theta_s \) and thus that \( \alpha < 2 (\beta < 4) \).
2.6 Other ISS Phenomena

DYNAMIC SPECTRA  When observed between, say, 100 and 1000 MHz, pulsar spectra show deep modulations in time and frequency (Rickett 1969). This phenomenon was early identified as diffractive scintillation (e.g. Scheuer 1968) and has since proved a rich source of information on the interstellar propagation process [for example, see CWB's important study of the dynamic spectra of many pulsars (Section 2.4)]. It is, however, only pulsars that have sufficiently small diameters to exhibit these effects.

The theory of DISS and the related phenomenon of pulse broadening are introduced in Appendix A. In the limit of fully saturated scintillations, there are independent variations over time scale $s_0/V$ and frequency scale $\delta v_d$ with a 100% modulation index. As the strength of scattering decreases, the refractive scintillations become more important and modulate the DISS in various ways. Refraction may be the cause of various apparently organized patterns that have been observed in pulsar dynamic spectra since the early observations of Ewing et al (1970). The observed phenomena are

1. drifting bands, in which a dynamic spectrum shows features drifting in frequency-time;
2. changes in the estimated frequency decorrelation bandwidth $\delta v_d$ over days to months;
3. criss-cross patterns, in which there are overlapping slopes of opposite sign; and
4. periodic patterns, in which there are repeating bands at an angle in the frequency-time plane, with from a few to tens of “fringes” modulating a single diffractive intensity burst.

Figure 5 shows examples of these phenomena. These features can be explained qualitatively by refractive steering of the diffractive pattern. However, the quantitative tests have not been conclusive, and scattering by specific deterministic structures is also possible. In particular, it is difficult to explain (d) on the basis of random RISS. Refraction is caused by phase structure on the spatial scale of the scattering disk ($s_r \approx L \theta_s$). Whereas intensity modulation is caused by a curvature in the wave front, a local refraction angle is caused by a linear phase gradient over the scale $s_r$.

**Drifting bands and variable frequency width**  The angles of refraction are given by $k \theta_{rx} = \partial \phi / \partial x$ and $k \theta_{ry} = \partial \phi / \partial y$, where the phase gradients are averaged over the scale $s_r$; typical values are given by Equations 2.8 and 2.9 for $0 < \alpha < 2$. Shishov (1974) gave a quantitative analysis in terms of refraction by a separate large-scale component of the medium. The angles are independent in the $x$ and $y$ directions and refractively shift (or steer) the diffractive pattern by distances $X \sim L \theta_{rx}$ and $Y \sim L \theta_{ry}$. The shifts are
Figure 5  Dynamic spectra from DISS of pulsars, showing a range of phenomena. Spectra, averaged over many pulses, are plotted versus time in a gray scale (plot density increasing linearly with amplitude). The spectra are for (a) PSR 1642 -0 3, (b) PSR 2016+ 28, (c) PSR 0832+26, and (d) PSR 1237 +25. Plots (a) and (b) are from Jodrell Bank (see Gupta et al 1988), and (c) and (d) are from Arecibo (J. M. Cordes, private communication; Wolszczan & Cordes 1987).

frequency dependent, approximately as $v^{-2}$, so that an intensity peak will appear to drift in frequency-time. An observer at a fixed site sees temporal changes due to motion of the pattern (at velocity $V$, which is chosen as the $x$ direction) and so sees a sloping feature (Figure 5a) with

$$\frac{dt}{dv} \sim \frac{(dX/dv)}{V} \sim \frac{(-2X/v)}{V} \sim -2L\theta_r/(vV).$$

Measurements of such drifting bands have been made by Smith & Wright (1985), who estimated the “drift rate” $dv/dt$ from observations of 32 pulsars and found $\theta_r/\theta_s$ to be consistent with a Kolmogorov spectrum. However, it is not a very precise estimate, since for each pulsar there was only a single estimate of $\theta_r$, which is expected to vary randomly about a zero mean over the refractive time scale. Systematic series of observations of
dynamic spectra have been made at Arecibo (J. M. Cordes & A. Wolszczan, private communication) and Jodrell Bank (Gupta et al 1988), but they are still being analyzed. A practical point is that in estimating the slope of a drifting band, it is more satisfactory to report $dt/dv$ rather than $dv/dt$, since according to Equation 2.10 it has an average of zero (and so an average $dv/dt$ of infinity). As mentioned above, steep-spectrum models predict larger $\theta_*/\theta_0$ ratios than does the Kolmogorov spectrum; thus, these observations can potentially discriminate between the models.

The same observations show fluctuations of the apparent decorrelation bandwidth $\delta v$ (e.g. Roberts & Ables 1982). Simple refraction has the effect of narrowing the frequency structure when $X$ or $Y$ are nonzero. A corollary is that observations of $\delta v$ will vary and will typically underestimate $\delta v_d$ (e.g. CPL). If one assumes normal statistics for $X$ and $Y$, the fractional bias and rms variation in $\delta v$ will be about $0.5 (\theta_*/\theta_0)$. Thus, the variations should be very pronounced for a steep power-law spectrum. For a Kolmogorov spectrum, they should be most noticeable near $\theta_*/\theta_0 \sim 1$ (which is near the transition from weak to strong scintillation) and decrease slowly as $u$ increases.

One of the parameters that can be estimated from the dynamic spectra is $\delta v_e$; others include the slope, the time scale, and the apparent average intensity. There are correlated variations expected between these parameters (see RNB for specific predictions). A preliminary analysis reported by Gupta et al (1988) showed significant deviations from this first-order theory, and more work is needed. For a discussion including a curvature as well as a gradient in the wave front, see CPL.

As can be seen in Figure 5c there can be opposite slopes overlapping in a single spectrum. The refractive shift interpretation implies two angles of arrival. It is not known whether this is compatible with a random distribution or whether instead it requires discrete density structures. The latter is an unattractive option, since the phenomenon is quite common, requiring widespread discrete structures. However, for propagation through an extended region of random density, it initially seems surprising to find only two angles of arrival contributing at a given point. Such effects seem more likely near $u \sim 1$, where focusing and interference are first occurring. Goodman et al (1987) proposed an interesting explanation for spectra with an inner scale; such spectra are strongly influenced by caustics, of which the commonest case is a "fold" with two dominant angles of arrival.

**Periodicities** Dramatic periodicities are evident in some dynamic spectra; examples can be seen in Figures 5b and 5d. This phenomenon has been a subject of increasing interest. Hewish (1980), Roberts & Ables (1982),
and Cordes & Wolszczan (1986) emphasized periodicities, which Hewish et al (1985) explained in terms of interference between waves arriving from two (or three) directions. They interpreted their observations as requiring ratios $\theta_f/\theta_s > 1$, which in turn implies a spectral exponent $\beta > 4 (\alpha > 2)$, a conclusion also reached by Roberts & Ables (1982). This seems to be in disagreement with much of the other evidence on spectral shape. An inner scale and the caustics of Goodman et al (1987) seem to be the best resolution of this inconsistency.

Wolszczan & Cordes (1987) observed a remarkable episode of periodic structure from PSR 1237 + 25 at 430 MHz. Figure 5d shows one of their spectra from 9 December 1986; observations 13 days later also showed fringes but with a larger period, and 19 days later the fringes had essentially disappeared. More than 10 fringes can be seen across a single diffractive scintle, which makes $\theta_f/\theta_s \sim 10$. Strong periodicities, also referred to as “multiple imaging events,” have been observed from other pulsars, but as yet there is no statistical analysis of their occurrence.

“Interferometry” In observing PSR 1237 + 25, Wolszczan & Cordes (1987) kept spectra resolved according to phase within the pulsar period. They found systematic shifts in the periodic spectra from different parts of the average pulse profile. This they interpreted as due to a lateral separation of the emitting regions across the pulse profile, which gives an exciting resolution of the pulsar emitting region. In reanalyzing their results, Cordes & Wolszczan (1988) concluded that emission is from regions near the “light cylinder” of the pulsar, in surprising disagreement with the standard hollow-cone pulsar models. They note that the apparent angular resolution is equivalent to an interferometer of 1 AU baseline. Wolszczan et al (1988) have reported a similar resolution of PSR 1133 + 16, except that the periodicity was predominantly in the time domain. Further applications of this technique should be very fruitful, although much observing time is needed to catch an event.

VELOCITY DETERMINATIONS The diffractive scintillation of pulsars has been used to extract a typical time scale $t_d$ as well as a typical frequency scale $\delta v_d$. The time scale is related to the spatial scale via the velocity $V$ of the diffraction pattern with respect to the Earth (i.e. $t_d \approx s_0/V$), and $s_0$ can be related to $\delta v_d$ and the pulsar distance $L$; thus a velocity estimate $V_{iss}$ is possible (e.g. Rickett 1970). Lyne & Smith (1982) used this technique to measure $V_{iss}$ for 20 pulsars and found statistically reasonable agreement with independent measurements of pulsar proper-motion velocities. Cordes (1986) made ISS observations of 65 pulsars and combined them with earlier data, giving a list of 71 pulsars with estimates of $\delta v_d$, $t_d$, and derived $V_{iss}$. He found good agreement in $V_{iss}$ with the values of Lyne &
Smith. The velocity estimate \( V_{\text{iss}} \) depends both on the velocity of the pulsar relative to the Earth and on the distribution and motion of scattering electrons along the line of sight, for which there is no precise model. Since pulsar velocities tend to dominate, Cordes was able to use his results to investigate the spatial distribution of pulsars and their velocities.

In a creative use of this technique, Lyne (1984) measured the apparent \( V_{\text{iss}} \) for the binary pulsar PSR 0655 + 64 as a function of phase in its binary orbit. He found that \( V_{\text{iss}} \) changed systematically around the orbit, providing a measure of the transverse speed; he thus estimated the systematic velocity, the orbital inclination, and the mass of the companion (<0.8 \( M_\odot \)). Dewey et al (1988) made similar observations of PSR 1855 + 09 and constrained the inclination of the orbit, which together with the known mass function gave the pulsar companion’s mass (~0.25 \( M_\odot \)). For PSR 1913 + 16, Dewey et al observed a variation in \( V_{\text{iss}} \) around the orbit but could not constrain the parameters significantly.

**TIMING VARIABILITY**  The intrinsic timing accuracy of most pulsars is about a milliperiod (see Cordes & Downs 1985), which is much greater than delay variations expected from the irregular densities of the ISM (R77). However, some millisecond pulsars now have timing accuracy better than a microsecond, and thus the ISM has become an important limit.

Rawley (1986) and Rawley et al (1988) report on the extraordinary timing stability of PSR 1937 + 21. From Arecibo observations at 1.4 and 2.5 GHz they estimated the contribution (~1 \( \mu \)s) due to varying dispersion delay. Cordes et al (1990) made time-of-arrival measurements at frequencies down to 0.32 GHz in order to investigate propagation effects. They too found very stable intrinsic behavior and slowly varying time differences between 0.43 and 1.4 GHz. The inferred dispersion variation showed only partial agreement with Rawley’s. The difference could be partly explained as an intrinsic frequency dependence of pulse shape or emission time, but the dispersion variations have become a little uncertain. However, Cordes et al note that other influences may be at work, such as time-of-arrival variations due to wander in angle of arrival, a phenomenon that scales more steeply with frequency (see also Blandford et al 1984). Clearly, this subject needs further work. Both Rickett (1988) and Cordes et al used the variable dispersion delay to infer the geometric phase (1 \( \mu \)s of delay corresponds to >10^3 radians), and they estimated \( D_\phi \) at scales up to 10^{12} m.

The ultimate timing accuracy from millisecond pulsars requires techniques for correcting dispersion variations (see Foster & Cordes 1988). At a lower level there are other propagation effects that cannot be corrected. Some effects depend on the method of dispersion removal used in the
receiver (e.g. postdetection, predetection, swept local oscillator). There is a diffractive effect that has not been considered, which could be important in timing PSR 1937+21. In the observations of Rawley et al (1988) and the swept local oscillator measurements of Biraud & Bourgois (1988), a total bandwidth of about 6–8 MHz was used. This includes only a few independent diffractive scintles in the band. Associated with each scintle there is an unavoidable timing uncertainty of \((2\pi v_d)^{-1}\) equal to the diffractive pulse broadening. This error should change on the time scale of the diffractive intensity scintillations. For a typical measurement reported by these two groups, this contributes \(\sim 0.1\) µs, which may become a significant timing limit, although it could be reduced by a larger bandwidth and integration time.

**FARADAY ROTATION FLUCTUATIONS** Multifrequency observations of a linearly polarized radio source allow the Faraday rotation measure (RM) to be determined. This quantity is proportional to the integral of the line-of-sight component of the magnetic field times the local electron density. Simonetti & Cordes (1988) summarized their measurements as the structure function of RM for separations from less than 1' to several degrees in the sky. It is not possible to separate the effects of variation in density and field, but Simonetti & Cordes found that irregularities in either one or the other extend to scales as large as 1 pc. In an interesting extension of this work Lazio et al (1990) measured RM variations across eight extended extragalactic radio sources using the VLA. The sources all lie in the heavily scattered Cygnus region of the Galaxy and showed significantly higher RM and RM variance than comparable sources at high galactic latitudes. Changes of RM by 40 rad m\(^{-2}\) over an arcminute were typical. Lazio et al concluded that they were seeing the effect of the density fluctuations responsible for the enhanced scattering. Using \(C_n^2\) estimates from VLBI of nearby sources and assuming a uniform field, they showed that their results are consistent with a density spectrum continuous from scales of \(10^7\) m to \(10^{15}–10^{17}\) m. Evidently, this is a powerful method for probing scales not previously accessible to radio techniques. However, interpretations in which magnetic rather than density fluctuations are responsible may also be possible, especially given the magnetic irregularities proposed for cosmic-ray confinement (Jokipii 1988).

### 3. PLASMA TURBULENCE IN THE ISM

The possibility of a turbulent plasma in the interstellar medium has emerged in the last two decades as a serious astrophysical question. The subject is beyond my ability to evaluate critically, and thus I summarize
the literature here with little critical evaluation. It is exciting to note that serious attempts are under way to understand the plasma physics of the density structures that cause ISS.

3.1 The Turbulence Hypothesis

Turbulence has long been discussed for the neutral interstellar gas, particularly concerning the need to dissipate turbulent energy before a gas cloud can collapse gravitationally towards "stardom." The theoretical questions of turbulence in the partially ionized interstellar gas clouds (the cool phase) have been addressed by various workers over the years (e.g. Kaplan 1966, pp. 103–22; Larson 1979, Fleck 1981). However, there is no satisfactory theory for turbulence in a magnetized plasma. Furthermore, there will be differing physics in the cool (∼100 K), warm (∼10^4 K), and hot (∼10^6 K) phases of the ISM (McKee & Ostriker 1977, McCray & Snow 1979). Lee & Jokipii (1976) suggested that the microscale density structures responsible for radio-wave scattering (scales of ∼10^9 m or smaller) are part of a spectrum of density irregularities that is continuous up to the scale of interstellar clouds or even to the thickness of the disk (10^{17}–10^{19} m). For power-law spectra (Equation A1) with β > 3, the variance of density (the integrated density spectrum) is determined both by C^2_k and by the low-wave-number cutoff. The associated rms density should then be less than or comparable to the local mean density. The results are consistent with a Kolmogorov spectrum and suggest a turbulent process with energy input from the parsec scales of the clouds and a turbulent cascade over 8–9 decades in wave number. However, there remain very serious problems with such an idea, and it is these that motivate the proposals for "steep" power-law spectra.

The first major question is whether we can even infer the presence of turbulence from density observations alone. The idea of turbulence in a neutral fluid is centered on the dynamics of the motion as a function of scale. Density perturbations are not directly involved in the dynamics, and so they provide no evidence for turbulence (or even randomness) in the velocity field or magnetic field. Higdon (1984) proposed that though the density structures are not central to the physics of the motion, they are nevertheless passive tracers. Similarly, Montgomery et al (1987) invoked an equation of state for the plasma to calculate weak density perturbations from pressure perturbations, obtaining a Kolmogorov form for the density spectrum. These interpretations support the idea of plasma turbulence down to the small radio-scattering scales. With this assumption, I now discuss the three parts of a turbulent spectrum: the power input range, the inertial (power-law) range, and the dissipation range.
3.2 Power Input

The power input at the largest scales could be random motion of cool clouds or H II regions, or expanding shock fronts from supernovae or large-scale stellar wind flows. The source could be one or a combination of these very different processes, and no consensus exists. In a recent paper Bykov (1988) calls for a mixture of these processes from scales up to 100 pc and a power input of $10^{-27}$ watts m$^{-3}$. Altunin (1981) argues against a cascade from such large scales and proposes instead that instabilities in shock propagation directly generate turbulence on smaller magnetoacoustic wavelengths of, say, $10^9$ m. Max et al (1988) also discuss a source mechanism at shocks. They discuss how a Fermi-type cosmic-ray acceleration mechanism may also generate large-amplitude Alfvén waves and associated density fluctuations. Spangler et al (1988) and Spangler & Cordes (1988) discuss the magnetohydrodynamic (MHD) waves at the Earth's solar wind bow shock, suggesting that similar wave generation may occur in interstellar shocks. Pimenov (1985) emphasizes the role of a shock passage in amplifying an existing spectrum of small-scale irregularities. It seems that shock generation sites cannot easily account for the diffuse scattering medium, but they might be responsible for the clumpy sites of strongly enhanced scattering.

3.3 Nonlinear Energy Exchange—The Inertial Range

Nonlinear interactions are necessary for a turbulent cascade from one scale to another. Possible wave modes include the Alfvén waves and fast and slow magnetoacoustic waves, but there is little theory for their nonlinear interaction. Matthaeus & Montgomery (1981) and Shebalin et al (1983) have made analytical and numerical assaults on plasma turbulence under the incompressible MHD approximation. They obtain predominantly two-dimensional turbulent flow normal to the local $B$ field; the velocity and magnetic field spectra take on the Kolmogorov form in two dimensions. On smaller scales, the local $B$ field includes the random components on all larger scales and so may not remain aligned with a large uniform field. Recently, Shebalin & Montgomery (1988) numerically modeled the effects of slight compressibility, finding results in agreement with those of Montgomery et al (1987). Note that Kraichnan (1965) has argued that the power-law exponent for a turbulent cascade in a hydro-magnetic plasma should be 3.5 rather than 3.67 (see also McIvor 1977, Bykov 1988, Tu 1988).

Several theoretical studies emphasize dissipation mechanisms, which might interrupt an energy cascade down to the small scales. For example,
McIvor (1977) and Cesarsky (1980) both discussed this question for the three phases of the medium. They concluded that important dissipation should occur at scales much larger than the radio-scattering scales, although they disagreed on some details; particularly for the hot phase, McIvor found thermal conduction effects to suppress any cascade, whereas Cesarsky argued that a cascade could exist from scales of $10^{17}$ to $10^{10}$ m. Zweibel et al (1988) considered waves generated in the hot ISM by the interaction of clouds and supernovae remnants, concluding that viscous damping precluded a turbulent cascade to small scales. They also noted, as had McIvor and Cesarsky, that collisions with neutrals would be an important damping mechanism in the partially ionized warm phase.

Higdon (1984, 1986) proposed that stationary constant-pressure density structures (entropy structures and tangential pressure balances) are convected and sheared by the turbulent velocity field and acquire the same spectrum as the velocity field. His arguments rely on analogies with turbulence in both a neutral gas and in an incompressible plasma but are a thoughtful consideration of this difficult problem. He suggests that the two-dimensional nature of the plasma turbulence (see above) may avoid the damping mechanisms discussed by McIvor and Cesarsky.

Jokipii (1988) has recently suggested that the magnetic irregularities that are involved in the confinement of cosmic rays in the Galaxy may be part of the same turbulent cascade. The diffusion of cosmic rays is determined by magnetic irregularities on a scale of the associated proton gyroradius, which for a magnetic field of 3 $\mu$gauss covers, say, $10^{10}$ to $10^{17}$ m. Jokipii's analysis for Fermi-acceleration of protons in strong interstellar shocks and a Kolmogorov spectrum of magnetic irregularities over this range of scales gives a proton spectrum in excellent agreement with the observations. This is an interesting new argument in support of turbulence at intermediate scales. Alfvénic fluctuations have been measured in the solar wind (Belcher & Davis 1971, Roberts et al 1987, Bavassano & Bruno 1989), and there has been a long-standing debate about whether these are noninteracting incompressible Alfvén waves generated in or near the Sun or whether they represent an input power at large scales and a nonlinear turbulent cascade to small scales (see reviews by Hollweg 1978, Barnes 1979). The picture that now emerges (e.g. Tu et al 1989) is a solar origin for Alfvénic turbulence with sufficient amplitude to cause nonlinear interaction between scales and associated density fluctuations. These studies certainly clarify the plasma physics of the fluctuations, and although the ISM lacks a spherical outflow, the suggestion of Alfvénic turbulence of sufficient amplitude to drive density fluctuations is in accord with many of the ideas posed for the ISM.
3.4 Dissipation

In a turbulent neutral gas the inertial range extends to an inner scale where viscous forces dissipate the turbulent energy. As discussed above, there has been no lack of dissipation mechanisms; indeed, the problem has been to avoid dissipation and find what wave modes might interact and transfer energy down to the small radio-scattering scales. Magnetohydrodynamic and magnetoacoustic modes can extend to length scales shorter than the mean free path for collisions. However, there should be no waves smaller than the gyroradius for the thermal protons. For the warm intercloud medium, which is most commonly proposed as the site for the turbulence, this scale is $\sim 10^4$–$10^5$ m, in agreement with the results of Spangler & Gwinn (1990) but much smaller than the $10^9$ m proposed by Coles et al (1987). Bieber et al (1988) discuss the dissipation scale for magnetic turbulence and its influence on cosmic-ray propagation. Harmon (1989) discusses density spectra and dissipation for Alfvén waves, associated with the inner scale, measured in the solar wind by Coles & Harmon (1989). These authors find enhanced density spectra just above a cutoff; if a similar enhancement exists in the ISM, there may be an associated enhancement in the focusing properties that might explain the double-imaging events in pulsar spectra.

A further theoretical difficulty surrounding the turbulence hypothesis is that active turbulence implies a power flux of, perhaps, $10^{-26}$ watts m$^{-3}$ from very large to very small scales, which is comparable to other heating and cooling rates for the ISM (see Spitzer 1978). In other words, the turbulence itself could be important in the energy balance of the plasma, supplying heat to the plasma at the dissipation scale and absorbing energy at the outer scale. Gibson (1988) has introduced the notion of fossil turbulence into this discussion. He argues, by analogy with ocean turbulence, that the density spectrum may be a “fossil” remnant of a turbulent process that is no longer active. The Kolmogorov density spectrum could persist well after the plasma has ceased being actively stirred by turbulent motion. This avoids the large steady power flux for active turbulence. A formal examination of this idea must address the lifetime of density structures in a collisionless plasma.

3.5 Sites for Enhanced Scattering

As discussed above, shock generation or amplification of plasma fluctuations has been proposed by several authors. Thus, supernova remnants have been an obvious candidate for the location of the regions of enhanced “turbulence.” Several groups of VLBI observers have looked for evidence
of an excess of source broadening around supernova remnants, where interstellar shocks are expected. The results have not been conclusive. An alternative site of enhanced scattering is proposed by Anantharamaiah & Narayan (1988) as the low-density outer envelopes of H II regions, particularly those responsible for galactic ridge recombination lines (see Anantharamaiah 1986, Kassim 1989). In addition, Spangler & Gwinn’s (1990) proposed inner scale is near the proton gyroradius for this medium. The enhanced scattering of NGC 6334B by a bipolar H II region (Moran et al 1990) is also a valuable clue.

3.6 **Nonturbulent Models**

Models that do not involve turbulence have also been proposed. A spectrum with $\beta \sim 4$ is consistent with a random superposition of discontinuities, as might be due to shock fronts; the constraint would be that the shock thickness was at least as small as the radio-scattering scales of $\sim 10^9$ m. In contrast, Hall (1980, 1981) has argued that a narrow spectral range in the density spectrum near $10^9$ m would result from the “mirror instability” in the hot phase of the ISM. Though ISS observations suggest a wider spectrum, it is interesting that this proposed peak is near the inner scale proposed at $10^9$ m.

4. **SUMMARY**

Interstellar scattering has now become important in several branches of astronomy and astrophysics, and the present situation can be summarized as follows:

1. **The bad news.** Scattering causes a seeing limitation in radio observations. This limitation was already evident for pulsars, but it is now clear that variation due to RISS is likely to be important for several classes of variable sources, particularly low-frequency variables and centimeter-wave flickering; RISS must also be included in the interpretation of sources small enough to vary intrinsically. However, even here the news is partly good, in that we now need fewer astrophysical explanations of extraordinary brightness temperatures. Interstellar scattering also broadens the image of many sources at low galactic latitude and provides a limit to VLBI resolution at low latitudes and long wavelengths (e.g. Dennison & Booth 1987).

2. **The good news.** By studying the flux variability of pulsars and extragalactic sources and the VLBI visibility curves, observers have new techniques for probing the ISM. The existence of a density spectrum covering many orders of magnitude in wave number is now established. There is a
hard question as to whether this spectrum is steeper or shallower than the exponent 4. There are conflicting interpretations of pulsar dynamic spectra, VLBI data, and image wander. Much of the evidence points to a shallower spectrum, with an inner-scale cutoff, which creates caustics. In spite of the great strides in wave propagation theory, particularly with regard to refractive effects, there are some unsolved questions. We lack quantitative interpretations of the periodic behaviors in pulsar dynamic spectra and in VLBI visibility curves obtained with self-calibration. It is likely that numerical simulation of these observations will resolve these questions.

Notable recent discoveries include the dramatic enhancements of scattering in the inner Galaxy and models for its distribution; the application of pulsar dynamic spectra to pulsar astronomy (binary orbit determination and resolution of a pulsar emitting region from episodes of “double imaging”); and the discrete propagation events, which may be due to deterministic structures.

3. The associated questions for theories of the ISM are now being addressed. The possibility of plasma turbulence in the various interstellar regions is under serious consideration. There are conflicting conclusions that support and deny a turbulent cascade over the proposed 8–10 orders of magnitude in scale. Location of the sites where “turbulence” is enhanced by a factor of $10^6$ will be an important clue. The fact that the solar wind has a similar density spectrum with the appearance of a turbulent cascade adds weight to the turbulence hypothesis. In situ solar wind measurements may help resolve the nature of the interstellar as well as interplanetary density irregularities.

APPENDIX A. WAVE PROPAGATION THEORY

The Density Spectrum

As in R77, I assume here that the plasma density irregularities may be characterized by a spatial power spectrum $P_{3\kappa}(\kappa)$, where $\kappa$ is the three-dimensional wave number. It is this function that we hope to determine from measurements of scintillation and scattering. The following form is useful:

$$P_{3\kappa}(\kappa) = C_{3\kappa}^2(r)(\kappa^2 + \kappa_{\text{outer}}^2)^{-3/2} \exp\left(\frac{\kappa^2}{\kappa_{\text{inner}}^2}\right).$$  \hspace{1cm} A1.

For wave numbers between $\kappa_{\text{outer}}$ and $\kappa_{\text{inner}}$, the power-law region is given by

$$P_{3\kappa}(\kappa) = C_{3\kappa}^2(r)\kappa^{-3}, \quad \kappa_{\text{outer}} \ll \kappa \ll \kappa_{\text{inner}}.$$  \hspace{1cm} A2.

For $\kappa < \kappa_{\text{outer}}$ the power spectrum saturates at a constant value; for $\kappa > \kappa_{\text{inner}}$ it falls rapidly to zero. The wave number $\kappa_{\text{outer}}$ is the reciprocal
of an outer scale, and $\kappa_{\text{inner}}$ is the reciprocal of an inner scale; between them they define an "inertial range" for the plasma turbulence. Unless specified otherwise, we assume the following simplified model: the wave numbers relevant to the scintillation are far removed from $\kappa_{\text{inner}}$ and $\kappa_{\text{outer}}$; the exponent $\beta$ is equal to $11/3$, corresponding to the Kolmogorov spectrum for turbulence in a neutral gas; the factor $C_N^2(r)$ is taken to vary slowly with position, and we define $\alpha = \beta - 2$. In the slab model $C_N^2$ it falls to zero outside the galactic disk of thickness $L_0$. The theory is stated for a general form of $P_3N(\kappa)$, where convenient. Good reviews of propagation through such a weakly scattering medium have been given by Prokhorov et al (1975) and Fante (1975, 1980).

**Second Moment of the Scattered Field**

Spatial deviations in the refractive index cause phase modulations as the waves travel through the ISM. The resulting wave front can be analyzed into a spectrum of plane waves, the width of which we call the scattering angle $\theta_s$. A point or plane-wave source thus suffers angular broadening. Consider a pair of antennas at vector positions $s$ and $s + \sigma$; the resulting interferometer fringes have electric field ($E$) amplitude and phase given by $E(s)E^*(s + \sigma)$. If this complex quantity is averaged sufficiently, we obtain the ensemble average visibility function, for which there is a simple and generally valid result:

$$\langle E(s)E^*(s + \sigma) \rangle = \exp \left[-0.5D_\phi(\sigma)\right].$$

This equation is independent of the position coordinate $s$ if the medium is statistically stationary. The quantity $D_\phi(\sigma)$ is the structure function of geometric phase—called the "wave structure function" in the optical literature; it is defined as

$$D_\phi(\sigma) = \langle [\phi(s) - \phi(s + \sigma)]^2 \rangle,$$

where $s = (x, y, L)$, $\sigma = (\xi, \eta, 0)$,

$$\phi(x, y, L) = r_e\lambda \int_0^L N_e(x, y, z) \, dz.$$

Here $\phi$ is the phase deviation calculated on a straight-line path ($z$ axis) from the source to the observer, assuming that the observing frequency is everywhere much greater than the local plasma frequency; $N_e(x, y, z)$ is the deviation in electron density from its mean; $\lambda$ is the wavelength (in the mean plasma density); and $r_e$ is the classical electron radius ($2.82 \times 10^{-15}$ m). Under astronomical conditions the pathlength $L \gg 1/\kappa_{\text{outer}}$, and the wave structure function can be related to the density spectrum:
\[ D_{\phi}(\sigma) = \int_{0}^{L} D'_{\phi}(z, \sigma) \, dz \quad \text{and} \quad D_{\phi,s}(\sigma) = \int_{0}^{L} D'_{\phi}(z, \sigma/L) \, dz. \quad \text{A6.} \]

The quantity \( D_{\phi} \) is basic to the wave propagation process, and different forms are needed for plane-wave (\( D_{\phi,p} \)) and spherical-wave sources (\( D_{\phi,s} \)). Its derivative \( D'_{\phi}(z, \sigma) \) in Equation A6 is related to the spectrum by

\[ D'_{\phi}(z, \sigma) = 4\pi\lambda^2 r_c^2 \int_{-\infty}^{\infty} \left[ 1 - \cos (\mathbf{\kappa} \cdot \mathbf{\sigma}) \right] P_N(\kappa_x, \kappa_y, \kappa_z = 0) \, d^2\mathbf{\kappa}. \quad \text{A7.} \]

Consider the spectral form of Equation A1 with \( 2 < \beta < 4 \) (or with \( \alpha = \beta - 2, 0 < \alpha < 2 \)) and \( \sigma < 1/\kappa_{\text{outer}} \):

\[ D'_{\phi}(z, \sigma) = \left[ \Gamma(1 - \alpha/2) \Gamma(1 + \alpha/2) \right] (8\pi^2/\alpha^2\lambda^2 r_c^2 C^2_N(z)^{\alpha^2})^{1/2} \quad \text{if} \ \sigma > 1/\kappa_{\text{inner}}, \quad \text{A8.} \]

\[ D'_{\phi}(z, \sigma) = \Gamma(1 - \alpha/2) \pi^{1/2} r_c^2 C^2_N(z)^{(\kappa_{\text{inner}})}^{1/2} \sigma^2 \quad \text{if} \ \sigma < 1/\kappa_{\text{inner}}. \quad \text{A9.} \]

Here \( D_{\phi} \) increases with \( \sigma \) from zero at the origin, and for \( \sigma > 1/\kappa_{\text{outer}} \) it saturates at a value equal to twice the phase variance; we define the outer scale by the value of \( \sigma \) at which it crosses half that value. For strongly scattering media, we can define a scale where \( D_{\phi}(\sigma) \) equals unity. This defines the field coherence scale \( s_0 \), which is the spatial separation across which an rms phase difference of 1 radian exists and also at which the visibility function (Equation A3) falls to \( e^{-1/2} \). The field coherence scale \( s_0 \) is often smaller than the outer scale by many orders of magnitude. By using Equations A6 and A8 or A9, it is straightforward to express \( D_{\phi}(\sigma) \approx (\sigma/s_0)^2 \) and so obtain \( s_0 \) in terms of the wavelength and the average of \( C^2_N \) over distance \( L \) for spherical- or plane-wave sources (see, for example, Coles et al 1987).

Since the visibility function in Equation A3 is a two-dimensional Fourier transform of the scattered source brightness distribution, the uncertainty relation between widths in the two domains suggests that we define the width of the scattered image (scattering angle) by \( \theta_s = 1/(ks_0) \) (i.e. Equation 2.4). This gives the width of an image averaged over very many realizations or over a spatial region much larger than the outer scale. The coherent integration times typical of a VLBI observation are not necessarily this long; in particular, if the density spectrum is “steep” (with \( \beta > 4 \)), the scattered image will wander by an angular extent much larger than its instantaneous angular size, which is accordingly much smaller than \( \theta_s \) defined by Equation 2.4. This situation can complicate the interpretation of VLBI observations (Section 2.5).
Weak Scattering

The scintillation index \((m)\) is the rms variation in intensity normalized by the mean intensity. In weak scintillation \((m \ll 1)\) there is a linear solution (Born solution) for the spectrum of the intensity variations as a Fresnel filter function times the phase spectrum for each scattering layer of thickness \(dz\). For a plane-wave source, the wave-number spectrum for the intensity variations is

\[
P_l(\kappa_x, \kappa_y) = 8\pi r_f^2 \lambda^2 \int_0^l \sin^2 (\kappa^2 L dz / 4\pi) P_N(\kappa_x, \kappa_y, \kappa_z = 0) \, dz.
\]

Here the medium extends from the observer to a distance \(L\). For a point source at distance \(L\), the \(z\) in the argument of the \(\sin^2\) function is replaced by \(\zeta(z, L) = z(L-z)/L\). For the density spectrum (Equation A1) with \(2 < \beta < 4\) and a uniform slab of scattering material, the resulting intensity spectrum increases with wave number up to an effective maximum wave number about equal to the reciprocal of the Fresnel radius \(r_f\) (Equation 2.1). The scintillation index is found as the square root of the integral of the appropriate \(P_l(\kappa_x, \kappa_y)\). When the index is substantially smaller than unity, the scintillations are weak and we find \(s_0 > r_f\). Weak scintillations are correlated over a wide frequency range, of the order of an octave [see R77 and Scott et al (1980) for the cross correlation of intensity variations in weak scintillations].

Strong Scattering

In strong scattering (i.e. a large scintillation index calculated from the Born solution), the linear relationship breaks down and no simple result is available. Then it also follows that \(s_0 < r_f\), which suggests the choice of \(r_f/s_0 = u\) as a parameter (Equation 2.5). For scattering concentrated in a thin “screen,” the intensity spectrum (arbitrary \(u\)) from a plane-wave source can be written (e.g. Equation 4.9 of Prokhorov et al 1975) as

\[
P_l(\kappa) = \int_{-\infty}^{\infty} \exp \{- F(r, \kappa L/k) + ik \cdot r\} \, d^2r / (4\pi^2),
\]

where

\[
F(r, t) = D_\phi(r) + D_\phi(t) - 0.5D_\phi(r + t) - 0.5D_\phi(r - t).
\]

For a spherical-wave source (at distance \(L\) from observer) seen through a screen (at distance \(z\) from the observer), Equation A11 applies with the effective distance \(\zeta(z, L)\) in place of \(L\).

For an extended medium, the solution is much more awkward and a
great deal of effort has been devoted to various regimes of approximation. Prokhorov et al (1975) gave approximations valid for high and low wave numbers (e.g. their Equations 4.40 and 4.41; see also Frehlich 1987). There is another interesting approximate solution (Uscinski 1982), that can be recognized as expressing the intensity spectrum as for an equivalent screen. Uscinski's Equation 102 for a plane-wave source, generalized to three dimensions, is identical to Equation A11, but with an equivalent function \( F_{eq} \) in place of \( F \):

\[
F_{eq}(r, t) = \int_0^L \{ D_\phi'(z, r) + D_\phi'(z, tz/L) \\ - 0.5D_\phi'(z, r + tz/L) - 0.5D_\phi'(z, r - tz/L) \} \, dz. \tag{A13}
\]

The equivalent-screen method for a point source in an extended medium has been presented by Uscinski et al (1982) and Macaskill (1983). Whitman & Beran (1985) explored the range of \( u \) for which the equivalent-screen solutions are valid, concluding that they apply for both weak and strong scattering, provided only that as a wave travels through the medium, large-phase fluctuations develop more rapidly than intensity fluctuations; this process can be described as extreme forward scatter. The solution is Equations A11 and A13 with a substitution in Equation A13 of \( L - z \) in place of \( z \) wherever \( z \) occurs in the second argument of \( D_\phi' \), and with the substitution of \( rz/L \) in place of \( r \).

Unfortunately, none of these expressions (Equations A11/A12 or A11/A13 or their spherical-wave equivalents) are particularly simple, and the form of the spatial spectrum of intensity is hard to visualize. We thus resort to approximation techniques.

In weak scattering we obtain Equation A10. In asymptotically strong scattering, the solutions take on a two-scale form. This can be seen from low- and high-wave-number expansions of Equation A11. The first-order high-wave-number expansion leads to the diffractive (small-scale) component. The first term of the low-wave-number expansion leads to the refractive (large-scale) component. It is described by a modified Equation A10, in which there is an additional cutoff at wave numbers above the reciprocal of the scattering disk size. The refractive term is actually of the second order compared with the first diffractive term, and the second diffractive term needs also to be considered, since it makes a contribution to the total variance equal to that of the first refractive term. This leads to the approximate result expressed in Equation 2.7. The expansions have been discussed by various authors since the review of Prokhorov et al (1975). For example, Codona et al (1986a,b) discuss the various techniques starting from a full wave treatment for spectra with exponents \( \beta < 4 \).
(They also show the equivalence of path integral and moment equation methods of analysis.)

Other groups have made useful alternative approximations to the diffractive and refractive regimes, two of which have concentrated on interstellar applications. Blandford & Narayan (1985) and RNB used a mixture of wave (for diffraction) and ray (for refraction) theories to examine the variability of many observable quantities. In a parallel investigation, CPL separated the phase perturbation of a screen into slowly varying (refractive) and rapidly varying (diffractive) components; by approximating the former by a Taylor expansion, they estimated the refractive and diffractive perturbations for many observable quantities. (Their separation of the phase into refractive and diffractive components remains an ad hoc procedure that has to be rechecked after the analysis is complete.) Similar results were obtained from both approaches and included many potentially observable parameters that had not been addressed in the full wave treatments. Both papers have tables of expressions relating observables to \( C_n^2 \), distance, and frequency. Although their numerical factors differ, the basic agreement of the two methods gives confidence in their results, and these studies have been used in several comparisons of observation and theory (e.g. for image wander by Gwinn et al. 1988).

The Caltech group had earlier demonstrated an interesting solution to the scattering problem for media with exponents \( \beta > 4 \) (Blandford & Narayan 1985, Goodman & Narayan 1985). These spectra have more energy in the low wave numbers as compared with the Kolmogorov spectrum and cause stronger refractive effects. The importance of this solution is that if such an electron density spectrum can explain the observations, there is no need to invoke turbulence in the interstellar plasma on the microscales responsible for interstellar scattering (see Section 3).

The following sections discuss the refractive and diffractive regimes of strong scattering for density spectra with \( \beta < 4 \). For steeper spectra the structure function has a square-law term that cancels in the second difference \( F \) (Equation A11), and thus a modified expansion method is needed (see below).

**Diffractive Scintillation**

The asymptotic high-wave-number expansion for the intensity spectrum \( P_I(\kappa) \) results in the Fourier transform of the square of the spatial covariance of the electric field, as in Equation A3. The resulting spatial scale for DISS is \( s_0 \) with a decorrelation time \( \sim s_0/V \). There is also an important decorrelation in frequency, \( \delta v_d \), which is typically in the range of 10 kHz to 10 MHz. There are thus independent diffractive scintles in frequency-time cells of \( \delta v_d \) by \( s_0/V \). The depth of modulation is 100%.
In the strong scattering limit, the intensity covariance, at position $s_1$ and frequency $v_1$, and position $s_2$ and frequency $v_2$, is approximated by the square of the second moment for the electric field. Consider the particular case of coincident receivers at different frequencies. The square of the second moment includes a term depending on the dispersion delay; as discussed by Lee & Jokipii (1975), a more useful approximation is obtained by removing the dispersive term. (I avoid calling it the refractive term in order to distinguish it from refractive scintillation.) However, there was no satisfactory derivation of how an intensity observation removes this dispersive term until the work of Codona et al (1986c). Their result is a modified dispersive term that arises from dispersion differences across the scattering disk. This term is important for frequency differences above about 0.1%. In typical interstellar observing conditions the term is unimportant, since diffractive decorrelation occurs over even narrower differences. Thus, as has been commonly assumed, diffractive decorrelation in frequency is governed by the square of the diffractive second moment versus frequency, which in turn is the temporal Fourier transform of the diffractive impulse response. Geometrical ray-path considerations give the typical broadening time for a pulse as

$$\tau_d = aL_\theta^2/c = 1/2\pi\delta v_d.$$  \hspace{1cm} A14.

Here $\theta_\theta$ is the apparent angular broadening at the observer, and $a = 0.5$ for a plane-wave incident on a thin screen at distance $L$ (and for a point source at distance $L$ with a screen halfway in between). For extended scattering, similar expressions result, with $L$ being the total pathlength and the factor $a$ depending on the distribution of irregularities. There is, regrettably, no simple exact result for the scattered pulse shape or the second moment of a point source in a uniform medium. Williamson (1974) made ray calculations for the scattered pulse shape in a uniform extended medium and showed that the rising edge of the pulse was particularly sensitive to the distribution along the line of sight of the scattering irregularities, which he assumed had a Gaussian spectrum. This is an area of research where fairly straightforward wave simulations could be done that would advance our ability to interpret the scattered profiles of pulsars. I use $a = 0.5$ in Equation A14 unless otherwise noted.

As explained above, the decorrelation function for diffractive scintillation is the Fourier transform of the scattered pulse shape, and so its width $\delta v_d$ is related to the pulse width $\tau_d$ as in Equation A14. This result has been tested by a number of authors comparing observations of pulse broadening with decorrelation of diffractive scintillations for the same pulsars. Evidently, its validity depends in detail on the definition of frequency and time scales; a half-power half-width in frequency and a $1/e$
width in time have often been used (see CWB, Slee et al 1980, Roberts & Ables 1982).

**Refractive Scintillation**

In asymptotically strong scattering, a low-wave-number expansion for the intensity spectrum yields the refractive scintillation spectrum. It is given by a modified Born solution, in which there is an additional cutoff at wave numbers above the reciprocal of the scattering-disk size—Equation A10 for \( P_s(\kappa) \) multiplied by \( \exp \left( - \int_0 D_z(z', \kappa z'/k) dz' \right) \) [see Frehlich (1987) for the spherical-wave form]. The refractive variance is given by integration over \( \kappa_x \) and \( \kappa_y \), and the Fourier transform gives the intensity correlation and hence the structure function needed to predict the depth of variation and time scales of RISS. It is straightforward to show that the bandwidth of RISS is about an octave, as for weak scintillation. The above results are only valid when the refractive variance \( m_r^2 \ll 1 \). This excludes the interesting region near \( u = 1 \), where we need numerical techniques.

**Inner-Scale Effects**

Most of the foregoing expressions apply for density spectra with \( \alpha < 2 \) with or without an inner-scale cutoff. However, the integrals then depend on the inner scale as well as on the strength of scattering. Coles et al (1987) showed how the presence of a cutoff in the spectrum can enhance the refractive variance over that of the Kolmogorov spectrum, adjusted to give the same strength of scattering. This provides a possible explanation of the RISS of pulsars, in which \( m_r \) is greater than the Kolmogorov value (Figure 2). The relative importance of the inner scale is determined by the ratio \( s_{\text{inner}}/L\theta_s \). The greatest enhancements in \( m_r \) occur where this ratio is unity.

Goodman et al (1987) went further in examining the “optics” near the focusing condition \( s_{\text{inner}}/L\theta_s = 1 \). Their analysis of caustics in this regime provides important insight. At distances somewhat beyond the focus there will be common occurrences of a “fold,” which corresponds to two distinct directions of arrival. This is a promising explanation of the double-imaging events (Figure 5d). In the limit of saturated scintillations there will be many directions of arrival, and the scintillations will take on a more stationary random appearance. The focusing condition for the inner scale of \( 10^9 \) m proposed by Coles et al (1987) probably corresponds to observations above 1 GHz at distances of 100–1000 pc and is represented by the heavy dashed line in Figure A1. This figure is for a uniform slab of scattering material, and large local deviations would be expected in the inner Galaxy. The focusing of a \( 10^6 \)-m inner scale would be less dramatic, since it is smaller than \( r_f \).
Figure A1  Lines of constant scale in “observing coordinates” for a uniform distribution of scattering material. Dash-dot lines are for constant diffractive scale $s_0$. Dashed lines are for constant refractive scale $s_r$. Solid lines are for constant Fresnel scale $s_f$. The heavy line divides weak from strong scattering. The heavy dashed line is where an inner scale of $10^9$ m would fill the scattering disk and so focus. Conditions near and to the left of this line might lead to double imaging. This plot only represents the diffuse scattering medium; large localized increases in scattering strength are to be expected, particularly toward the inner part of the Galaxy.

**Steep Spectra**

Goodman & Narayan (1985) have analyzed scintillation from a screen with $\beta > 4$ in Equation A1; the spectrum is then said to be steep. There are important modifications to Equations A8 and A9. Ignoring the effect of an inner scale, the dominant term for $D_\phi(z, \sigma)$ at small $\sigma$ is a square law. This gives $D_\phi(\sigma) \approx (\sigma/s_0)^2$, with the $s_0$ determined by the outer scale. The geometrical phase is dominated by linear gradients, and in terms of the scattered image the angle $1/k s_0$ corresponds to image wander ($\theta_i \sim 1/k s_0$). The width of the instantaneous scattered image comes from deviations of $D_\phi(\sigma)$ from a square law. Equations A11 and A12 still describe the intensity spectrum. However, in Equation A12 $F$ is completely independent of square-law terms in $D_\phi$. The next term in a $D_\phi$ expansion can be written as $-(\sigma/s_\alpha)^{\alpha}$; for $4 > \alpha > 2$ the scale $s_\alpha$ is independent of the outer scale, depending only on $C_N^2$ (as for $2 > \alpha > 0$).

The scale $s_\alpha$ determines both the development of intensity scintillations
and the instantaneous width of the scattered image. Evaluation of the integral in Equation A11 gives a “two-scale” intensity spectrum, but the negative sign and the fact that \( \alpha > 2 \) changes the behavior. The results can be understood by considering a modified \( D_\phi \), namely

\[
D_\phi = (\sigma/s_s)^2 - D_\phi(\sigma) \approx (\sigma/s_s)^2, \tag{A15}
\]

which represents deviations from linear phase gradients. The instantaneous scattering disk (of scale \( s_s \)) is determined by the interference condition that \( s_s \approx L \theta(s) \), in which \( \theta(s) \) is the scattering angle determined from the modified phase gradient averaged over scale \( s \) {i.e. \( k \theta(s) = [D_\phi(s)]^{0.5}/s \). Inserting the form Equation A15 gives}

\[
s_s \approx r_l/(s_s)^{(4-\alpha)}.
\]

The instantaneous scattering angle is \( \theta_s = s_s/L \), and the diffractive scale \( s_d \) is \( 1/k \theta_s \). This result was originally found by Blandford & Narayan (1985) and refined in several subsequent papers.

Literature Cited


Rickett, B. J. 1988. See Cordes et al 1988a, pp. 2–16
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